# Elliptical diagnostics of stratospheric polar vortices

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#### SUMMARY

Diagnostics based on the spatial moments of isopleths of a long-lived tracer, or of potential vorticity, are presented that enable the structure and evolution of stratospheric polar vortices to be concisely summarized and quantified. The area, centre, aspect ratio and orientation of the 'equivalent ellipse' of the vortex, on an isentropic surface, are defined using the second- and lower-order spatial moments of contours within the vortex-edge region. By examining the variations of these 'elliptical' diagnostics with time and altitude, the temporal evolution and vertical structure of the polar vortices can be quantified. The usefulness of the diagnostics is assessed by examining nitrous oxide data from the Geophysical Fluid Dynamics Laboratory 'SKYHI' general-circulation model. The diagnostics show, and quantify, several differences between the Arctic and Antarctic vortices in the SKYHI model. The Arctic vortex moves further off the pole, is generally more elongated, and has a more complicated vertical structure than the Antarctic vortex (with larger variations of both the vortex centre and elongation with height). The elliptical diagnostics also identify the occurrence of large-scale Rossby-wave breaking events, both at the vortex edge and in the subtropics, in the model.

KEYWORDS: Isentropic analysis Rossby waves Tracer transport Vortex structure Wave breaking

## 1. INTRODUCTION

Tracer transport in the wintertime stratosphere is dominated by a strong, cyclonic, polar vortex. There is reduced meridional transport through the edge of these vortices, and there are steep meridional gradients of quasi-conservative tracers (e.g. potential vorticity (PV) and long-lived chemical species) at the vortex edge (e.g. McIntyre and Palmer 1983, 1984; Leovy et al. 1985). The vortex edge is distorted by upward-propagating Rossby waves, and during large-amplitude events (so called Rossby-wave breaking events) the edge is irreversibly deformed and vortex air is mixed into middle latitudes (McIntyre and Palmer 1983, 1984). The vortex evolution and the accompanying tracer transport is clearly shown in isentropic maps of quasi-conservative tracers, but these maps provide only qualitative information. However, a commonly used quantitative diagnostic which is based on these maps is the area enclosed by isopleths of quasi-conservative tracers on isentropic surfaces. This area diagnostic enables the formation and breakdown of the polar vortices and the occurrence of large-wave breaking events to be quantified (e.g. Butchart and Remsberg 1986; Baldwin and Holton 1988; O'Neill and Pope 1990; Manney et al. 1995; Nakamura 1995). The area enclosed by a contour corresponds to the zeroth order spatial moment of the contour. By calculating higher-order moments it is possible to quantify deformations to the contour. In this paper a set of diagnostics based on these higher-order spatial moments is presented, and the ability of these diagnostics to describe and quantify the structure and evolution of stratospheric polar vortices assessed. Spatial moments of vorticity isopleths have been used extensively in studies of twodimensional vortex dynamics to calculate constants of the motion, to form reduced models of the flow, and to diagnose the flow in numerical simulations (e.g. Melander et al. 1986, 1988; Dritschel 1986, 1993; Legras and Dritschel 1993). Here the same spatial moments are used to diagnose the evolution of stratospheric polar vortices. As outlined in the next section, the zeroth order spatial moment of a contour is the area enclosed by the contour, the first-order moment defines the centroid of the contour, and the second- (and higher) order moments define elliptical (and higher) deformations to the contour. Hence, the second- and

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lower-order moments can be used to define the area, centre, aspect ratio, and orientation of an equivalent elliptical contour. These diagnostics (referred to as 'elliptical diagnostics') can then be used to quantify the movement and elongation of a vortex.

To assess the ability of these elliptical diagnostics to quantify the structure and evolution of stratospheric polar vortices, data from the Geophysical Fluid Dynamics Laboratory 'SKYHI' general-circulation model (GCM) are examined. In particular, nitrous oxide (N<sub>2</sub>O) data from this GCM are used to define the polar vortices. N<sub>2</sub>O is a tropospheric source gas, with a long photochemical lifetime in the lower and middle stratosphere, and has been used in many studies to examine polar vortex dynamics and stratospheric transport in the real stratosphere (e.g. Hartmann *et al.* 1989; Schoeberl *et al.* 1992; Manney *et al.* 1994), and in the SKYHI model (e.g. Strahan and Mahlman 1994; Yang 1995; Eluszkiewicz *et al.* 1996). The data analysed here are from the high-resolution 1° latitude  $\times 1.2^{\circ}$  longitude calculation which produced a realistic simulation of both the northern and southern hemispheres, although it suffers from the usual cold-pole problem in the southern winter (Mahlman and Umscheid 1987; Strahan and Mahlman 1994).

Note that the diagnostics presented in this paper can be applied to fields of any long-lived tracers or PV, either from observations or from models. SKYHI  $N_2O$  was used in this assessment study solely because the data set was readily available. Preliminary examination of diagnostics applied to analysed PV show similar results to those presented in this paper.

Using the elliptical diagnostics to analyse the stratospheric polar vortices (and hence the structure of the wintertime stratosphere) provides an alternative (semi-Lagrangian) approach to the traditional Eulerian framework of analysing zonal wave structure (e.g. Randel 1988; Shiotani *et al.* 1990; Manney *et al.* 1991). In section 6 (and appendix) the elliptical diagnostics are compared with the results of a Fourier decomposition of the zonal wave structure in the N<sub>2</sub>O data. In particular, whether or not the location and elongation of the polar vortex can be inferred from the amplitude and phase of zonal wave numbers 1 and 2 (at a fixed latitude) is examined.

The spatial moments and diagnostics used in this paper are defined in the next section. In section 3 these diagnostics are applied to the  $N_2O$  on the 700 K isentropic surface during a 21 day period in northern winter. The ability of these diagnostics to describe and quantify the structure and evolution of the polar vortex is assessed, and the sensitivity of the diagnostics to the value of  $N_2O$  used to represent the vortex edge is examined. The diagnostics are then used in sections 4 and 5 to examine the temporal evolution and vertical structure of the Arctic and Antarctic vortices. The relationship between the elliptical diagnostics and diagnostics from a Fourier analysis is examined in section 6. The final section consists of some concluding remarks and a discussion on future work.

# 2. Elliptical diagnostics

In this section the diagnostics used in subsequent sections to examine the evolution of the stratospheric polar vortices are defined. These diagnostics have been used previously in studies of two-dimensional vortex dynamics.

First consider the spatial moments of a planar region  $\mathfrak{D}$  which is bounded by contour (or contours)  $\mathscr{C}$ . Following Legras and Dritschel (1993) and Dritschel (1993) we define the *m*-th complex moment, relative to a point  $z_0$ , as

$$\alpha_m(z_0) = \int \int_{\mathfrak{D}} (z - z_0) \, \mathrm{d}x \, \mathrm{d}y, \quad (m = 0, \, 1, \, \ldots) \tag{1a}$$

$$=\frac{i}{2m+1}\oint_{\mathscr{C}} (z-z_0)^{m+1} dz^*,$$
 (1b)

where x and y are the planar co-ordinates, z = x + iy,  $z^*$  is the complex conjugate of z, and Green's theorem has been used to form the second equation. Note that the complex moments  $\alpha_m$  are a subset of all spatial moments

$$J_{mn} = \int \int_{\mathfrak{B}} x^m y^n \, \mathrm{d}x \, \mathrm{d}y,$$

but the set of moments  $\alpha_m$  is all that is required to define the contour  $\mathscr{C}$  completely (Dritschel 1993).

The zeroth order moment  $\alpha_0$  is purely real, independent of  $z_0$ , and equals the area, A, enclosed by the contour  $\mathscr{C}$ , i.e.

$$\alpha_0 = A. \tag{2}$$

The first-order moment  $\alpha_1$  is imaginary and defines the centroid  $z_c = x_c + iy_c$  of the contour  $\mathscr{C}$ :

$$\alpha_1(0) = A z_c. \tag{3}$$

The second-order, and higher-order, moments are measures of elliptical, and higher-order, deformations to the contour  $\mathcal{C}$ . To interpret these higher-order moments geometrically it is convenient to define them relative to the centroid  $z_c$ . The second-order moment of an elliptical contour is (Melander *et al.* 1986; Dritschel 1993):

$$\alpha_2(z_c) = \frac{A^2}{4\pi} \left( \delta - \frac{1}{\delta} \right) e^{-2i\theta}, \qquad (4)$$

where  $\delta$  is the aspect ratio (defined as semi-major axis divided by semi-minor axis;  $\delta \ge 1$ ) and  $\theta$  is the angle of the semi-major axis to the x-axis. By calculating  $\alpha_2$  for an arbitrary contour  $\mathscr{C}$  it is therefore possible to determine from (4) the aspect ratio and orientation of its 'equivalent ellipse', i.e.

$$\delta = |P| + (1 + |P|^2)^{1/2}$$
(5a)

$$- [x] + (1 + [x]) ,$$
 (54)

and

$$\theta = -\frac{1}{2} \tan^{-1} \left( \frac{P_{\rm i}}{P_{\rm r}} \right), \tag{5b}$$

where

$$P = P_{\rm r} + \mathrm{i}P_{\rm i} = 2\pi\alpha_2(z_{\rm c})/A^2.$$

Thus from Eqs. (1) to (5) it is possible to determine the area A, centroid  $z_c$ , aspect ratio  $\delta$  and orientation  $\theta$  of the equivalent ellipse of a given contour  $\mathscr{C}$  (region  $\mathfrak{D}$ ). These quantities are what we refer to as the 'elliptical diagnostics'. Note that only second- and lower-order moments are used in Eqs. (2) to (5); using higher-order moments it is possible to diagnose higher-order (smaller scale) deformations to contours, see Legras and Dritschel (1993).

When interpreting the above elliptical diagnostics (in particular  $\delta$  and  $\theta$ ) it is useful to have a measure of the closeness of the shape of the contour to an ellipse. One such measure is the mean-square departure of the contour from an ellipse (D. Dritschel 1996, private communication)

$$\epsilon = \frac{\pi}{2A^2} \oint_{\mathscr{C}} [(1+|P|^2)^{1/2} r^2 - \Re e\{P^* \hat{z}^2\}]^2 \,\mathrm{d}\phi - 1, \tag{6}$$

where  $\hat{z} = z - z_c$ ,  $r = |\hat{z}|$ , and  $d\theta = (x dy - y dx)/r^2$ .  $\epsilon$  is positive definite and increases from zero as the contour  $\mathcal{C}$  deviates from an ellipse.

Here spherical data are analysed, and to apply the above planar equations the spherical data are mapped onto the plane using the polar stereographic projection

$$x = \frac{\cos \lambda \cos \varphi}{1 \pm \sin \varphi}, \quad y = \frac{\pm \sin \lambda \cos \varphi}{1 \pm \sin \varphi},$$

where  $\varphi$  is latitude,  $\lambda$  is longitude, and the positive (negative) sign is used in the northern (southern) hemisphere. The resulting A and  $z_c$  are then transformed into the equivalent spherical quantities using the reverse mapping.

Note that the spatial moments (and elliptical diagnostics) can be calculated directly in spherical coordinates, see Dritschel (1993). The resulting expressions are more complicated than their planar equivalents, and it was decided to use the simpler planar formulae in this paper. However, some of the calculations have been repeated using the spherical moments, and the results are very similar to those obtained using the planar moments together with the above mapping procedure.

As discussed in the introduction, N<sub>2</sub>O is a good tracer of stratospheric fluid motion. In this paper isentropic N<sub>2</sub>O isopleths are used to define the edges of the polar vortices, and the area A, centre ( $\varphi_c$ ,  $\lambda_c$ ), aspect ratio  $\delta$  and orientation  $\theta$  of the vortex are calculated from these isopleths. These diagnostics are then used to examine the evolution and structure of the polar vortices. The sensitivity of the elliptical diagnostics to the choice of N<sub>2</sub>O isopleth is discussed in the next section.

As can be seen from Eq. (1) the spatial moments can be calculated either by area integrals or contour integrals. In this analysis the latter are used because the specified  $N_2O$  contour may not be simply connected, and using contours in the analysis makes it easy to calculate the moments of individual (simply-connected) regions. As discussed later, using all contours or only the one encompassing the largest area can have an impact on the resulting diagnostics. Also, the Arctic vortex is known to split in two during some major warmings (although this does not happen in the SKYHI simulation considered) so it may be desirable to calculate the moments of individual vortex fragments during such an event.

# 3. SENSITIVITY TESTS

The usefulness and sensitivity of the elliptical diagnostics are now examined by considering the evolution of the simulated Arctic vortex on the 700 K isentropic surface during the period 18 December to 7 January.

Figure 1 shows a series of polar stereographic maps of  $N_2O$  on the 700 K surface during this period. There is a strong polar vortex with steep  $N_2O$  gradients at its edge, and weak gradients both inside the vortex and in middle latitudes. There are also steep gradients at the tropical edge of the surf zone. Throughout the period the vortex is centred off the pole over northern Europe. The vortex is nearly circular at the beginning of the period (18 December) and then elongates and rotates eastward (22 to 30 December). Tongues of air from the edge of the vortex are then drawn into the middle latitudes (30 December to 3 January) in planetary-scale Rossby-wave breaking events (McIntyre and Palmer 1983, 1984). These tongues are stretched and wrapped into middle latitudes, and (at the resolution of the  $N_2O$  field) become detached from the vortex. After this breaking event the vortex reverts to a nearly circular shape (7 January) and is noticeably smaller than at the beginning of the period.

First consider the evolution of the vortex as defined by the  $N_2O = 60$  parts per billion (10<sup>9</sup>) (p.p.b.) contour. This contour is within the region of steep gradients at the edge of the

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Figure 1. N<sub>2</sub>O on the 700 K isentropic surface for the period 18 December to 7 January. Contour interval is 20 p.p.b. and minimum value shown is 30 p.p.b. Maps are polar stereographic projections; outer edge is at 15°N, and dashed circles are at 30°N and 60°N. Dates are given in *dd/mm* format.

vortex (the 'vortex-edge region'). Using the procedure outlined in the previous section the elliptical diagnostics of this contour are calculated. Figure 2 shows these ellipses together with the actual N<sub>2</sub>O contour. There is good agreement between the equivalent ellipses and the actual contours, except during the wave breaking event in the middle of the period. The time series of the elliptical diagnostics are shown, as the solid curves, in Fig. 3. Note that

the area A is expressed as the lattitude of a zonal circle which encloses the same area (the so-called equivalent latitude  $\varphi_E$ ). These curves show the features discussed above: during the 21 day period the size of the vortex decreases (Fig. 3(a)), the vortex is centred off the pole over northern Europe (Figs. 3(b) and (c)), the vortex is roughly circular ( $\delta \approx 1$ ) at the beginning and end of the period but is elongated during the middle of the period (Fig. 3(d)), and the vortex rotates eastward (Fig. 3(e)). Not only do these parameters show the large-scale features shown in the N<sub>2</sub>O maps concisely, but they also enable these features to be quantified; for example, it is possible to determine the average position of the vortex centre (80°N, 20°E), the maximum equatorward position ( $\varphi_c = 76^\circ$ N) and elongation ( $\delta = 2.6$ ) of the vortex, and the average rotation rate of the vortex (period = 20 days).

As shown in Fig. 2, the elliptical fit to the contour is poorest in the middle of the period. This is quantified in the time series of the mean-square departure  $\epsilon$ , where there is a rapid increase during the wave breaking event. This suggests that  $\delta$  and  $\theta$  may be less appropriate as diagnostics of the contour during the period. However, Fig. 2 shows that even though  $\epsilon$  increases there is still a reasonable fit between the ellipse and the actual N<sub>2</sub>O contour, and  $\delta$  and  $\theta$  still provide a good measure of the elongation and orientation of the N<sub>2</sub>O contour during this wave breaking event.

The above calculation has shown that the elliptical diagnostics enable the evolution of the vortex (as defined by the  $N_2O = 60$  p.p.b. contour) to be easily displayed and



Figure 2. Plots of the  $N_2O = 60$  p.p.b. contour and the elliptical fit to this contour for the period shown in Fig. 1. The solid lines inside the ellipses correspond to the major and minor axes of the ellipse. Map projections and date format are as in Fig. 1.

quantified. However, it is not clear how sensitive the results are to the value of  $N_2O$  used in the calculation. To investigate this sensitivity the equivalent ellipses for the  $N_2O =$ (20, 30, ..., 150) p.p.b. contours have been calculated. Maps of the resulting ellipses for three days are shown in Fig. 4; for  $N_2O$  contours in the vortex-edge region (40 to 80 p.p.b.) the ellipses have similar shape, with the poorest agreement occurring during the wave breaking event. Note that the evolution of the ellipses during this event is very similar to simulations of vortex 'stripping' using the elliptical model of Legras and Dritschel (1991) (D. Dritschel 1996, private communication). The variations of the diagnostics for 40 p.p.b.  $\leq N_2O \leq 80$  p.p.b. are shown in Fig. 3. The area is sensitive to the value of  $N_2O$  but shows the same temporal variation for all contours. The variation in the other diagnostics is much smaller, and the temporal evolution and mean values for all contours are very similar. Hence this comparison indicates that the resulting diagnostics are not very sensitive to the value of  $N_2O$  used in the analysis.

In the above calculations only the largest  $N_2O$  contour has been used to determine the spatial moments; i.e. the small 'blobs' of  $N_2O < 60$  p.p.b., outside the largest contour in Fig. 2, are not used in the calculation of the moments for  $N_2O = 60$  p.p.b. The reason for this is to determine the best elliptical fit to the polar vortex; the inclusion of small regions outside the vortex can have a large effect on the resulting fit (especially if the regions are a large distance from the vortex centre). As an example, Fig. 5 shows the elliptical fit to the  $N_2O = 90$  p.p.b. contour on 3 January when all contours are used (dashed ellipse) and when only the contour with largest area is used (solid ellipse). The fit to the vortex is much better when only the contour with largest area is used. Note that when the above procedure is applied to assimilation PV there is a second reason for the exclusion of small outer contours: many of the 'blobs' of PV in middle latitudes are thought to be artifacts of the assimilation procedure (e.g. Plumb *et al.* 1994; Carver *et al.* 1994).



Figure 3. Temporal variation of (a)  $\varphi_E$ , (b)  $\varphi_c$ , (c)  $\lambda_c$ , (d)  $\delta$ , (e)  $\theta$  and (f)  $\epsilon$  for the period shown in Fig. 1.  $\varphi_E$  is the latitude of a zonal circle with area A. Quantities are shown for N<sub>2</sub>O = 40 (dashed), 50 (dash-dot), 60 (solid), 70 (dash-dot-dot-dot), and 80 (long dashes) p.p.b. In (a)  $\varphi_E$  is shown for N<sub>2</sub>O = (20, 30, ..., 150) p.p.b. Dates are given in *dd/mm* format. See text for further explanation.



Figure 4. Elliptical fits to  $N_2O = (20, 30, ..., 150)$  p.p.b. contours for 18 December, 30 December and 7 January. Map projection as in Fig. 1.



Figure 5. Plots of  $N_2O = 90$  p.p.b. contour plus elliptical fit using all contours (dashed ellipse), and only contour with largest area (solid ellipse), for 3 January.

During the above wave breaking event the vortex is highly distorted. This wave breaking event is one of the largest that occurred during the SYKHI simulation. For the periods other than these large-scale wave breaking events the vortex is less distorted, there is very little sensitivity to the value of N<sub>2</sub>O (for N<sub>2</sub>O within the vortex-edge region), and there is much better agreement between the equivalent ellipse and the actual N<sub>2</sub>O contour, i.e. smaller  $\epsilon$  (see Figs. 6 and 10). Actually, as variations in the diagnostics for different N<sub>2</sub>O values and rapid changes in  $\epsilon$  occur when tongues of air are stripped from the vortex edge, the occurrence of these features provides indicators of large-scale wave breaking events. This is highlighted in the next section.

# 4. SEASONAL EVOLUTION IN THE MIDDLE STRATOSPHERE

The evolution of each polar vortex on the 700 K isentropic surface is now examined over the annual cycle, using the elliptical diagnostics.

# (a) Evolution of the Arctic vortex

Figure 6 shows the time series of the diagnostics for the  $N_2O = 60$  p.p.b. contour from 1 September to 31 May (autumn to spring). The evolution of the area for a range of  $N_2O$  values is plotted in Fig. 6(a), and shows the formation of the vortex in September–October and a slow decrease in vortex size from November to May. Note that during the period when there is a strong vortex (November–April), the  $N_2O = 60$  p.p.b. contour is within



Figure 6. Temporal variation of (a)  $\varphi_E$ , (b)  $\varphi_c$ , (c)  $\lambda_c$ , (d)  $\delta$ , (e)  $\theta$  and (f)  $\epsilon$  for N<sub>2</sub>O = 60 p.p.b. contour on 700 K surface in the northern hemisphere, from 1 September to 31 May. The longitude of the vortex centre  $\lambda_c$  is not plotted if the latitude of the centre  $\varphi_c$  is greater than 89°N. Similarly,  $\theta$  is not plotted if  $\delta < 1.1$ . Horizontal dashed lines correspond to average values. Also shown in (a) is  $\varphi_E$  for N<sub>2</sub>O = (20, 30, ..., 150) p.p.b. Note that tick marks on the abscissa are at the middle of each month. See text for further explanation.

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Figure 7. Maps of N<sub>2</sub>O on the 700 K surface for 20 October, 8 February, and 21 February. N<sub>2</sub>O = (40, 60, 80) p.p.b. contours are plotted. Map projection is the same as in Fig. 1.

the vortex-edge region (as defined by the region of steep gradients). The diagnostics for other values of N<sub>2</sub>O within the edge region (40 p.p.b.  $\leq$  N<sub>2</sub>O  $\leq$  80 p.p.b.) are very similar to those shown in Fig. 6, although there are some differences during large-wave breaking events.

The vortex is generally centred well off the pole (mean latitude  $\bar{\varphi}_c = 81^\circ$ N), and in the 20°W–90°E region (mean longitude  $\bar{\lambda}_c = 28^\circ$ E). This is consistent with observations that show that in the middle and upper stratosphere the Arctic vortex is usually centred in this region, with a strong anticyclone over the Aleutian Islands (e.g. Leovy *et al.* 1985; Fairlie and O'Neill 1988; O'Neill *et al.* 1994). In February, April and May there are extreme events where the vortex centre moves equatorward of 75°N. The rapid equatorward shift of the centre in late May corresponds to the breakdown of the vortex; during and after this period there is not a coherent vortex, and there are large variations between the diagnostics for different values of N<sub>2</sub>O.

The aspect ratio of the vortex varies considerably during the year (Fig. 6(d));  $\delta$  varies between 1.0 and 3.0, and there are many periods of a few days duration when  $\delta$  exceeds 2. During several of these events there is also an increase in the mean-square departure  $\epsilon$ ,

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see Fig. 6(f). The analysis of the late-December event in the previous section suggests that the occurrence of a rapid increase in  $\epsilon$  indicates the occurrence of a large-scale Rossbywave breaking event. Figure 7 shows N<sub>2</sub>O maps for three of the events in which both  $\delta$ and  $\epsilon$  are large: 20 October, 8 February and 21 February. During these events, and the other events with  $\delta$  and  $\epsilon$  large (not shown), large-scale wave breaking is occurring with vortex air transported into middle latitudes. This provides further support for the use of a rapid increase in  $\epsilon$  as an indicator of the occurrence of large-scale wave breaking. Note that associated with the three largest events (in terms of elongation of the vortex) there is a noticeable decrease in area of vortex (Fig. 6(a)) and increased variability between the diagnostics for different values of N<sub>2</sub>O (not shown).

There are also several events where  $\delta$  is large but  $\epsilon$  is small, e.g. 2 February, 28 February and 20 March. As Fig. 8 shows, during these events the vortex is elongated but there is no wave breaking at the vortex edge (N<sub>2</sub>O  $\approx$  60 p.p.b.). Hence there are events where the vortex is very distorted (elongated) but wave breaking (irreversible transport of vortex air) does not occur.

Rossby-wave breaking events not only transport vortex air into middle latitudes but they can also transport tropical air into middle latitudes (e.g. Leovy *et al.* 1985; Randel *et al.* 1993). It has been noted by several studies that these 'tropical' wave breaking events generally occur when the vortex is displaced off the pole: (e.g. Leovy *et al.* 1985; Waugh

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Figure 8. As in Fig. 7 except for 2 February, 28 February, and 20 March.



Figure 9. As in Fig. 7 except for 23 November, 1 December, and 20 December; and for  $N_2O = 60$  and 160 p.p.b.

1993; Norton 1994; Polvani *et al.* 1995). Hence, the time series of  $\varphi_c$  may provide an indicator of the occurrence of tropical transport events, i.e. tropical breaking events are expected to occur when  $\varphi_c$  is equatorward of a critical latitude. Indeed, examination of maps of N<sub>2</sub>O for periods when the vortex centre is well off the pole ( $\varphi_c < 77^\circ$ N) do show tropical transport events occurring; for example Fig. 9 shows N<sub>2</sub>O contours, at the edge of the vortex and the tropics, for three of these events.

# (b) Comparison of evolutions of Antarctic and Arctic vortices

Figure 10 shows the time series of the diagnostics for  $N_2O = 60$  p.p.b. in the southern hemisphere, and should be compared with Fig. 6. The discontinuity at 6 June is because the N<sub>2</sub>O data are from a simulation which ran from 6 June to 5 June the following year, and so the data from 5 June and 6 June are from different years.

The evolution of the Antarctic vortex has many features similar to that of the Arctic vortex: increase of area in autumn and decrease in spring, variability in position and elongation, and centred in a preferred longitude range. But there are some significant differences: the Antarctic vortex is larger, it is less displaced off the pole ( $\bar{\varphi} = 85^{\circ}$ S compared with 81°N), it is less elongated ( $\bar{\delta} = 1.3$  compared with 1.5), and there are smaller extreme events in the southern hemisphere (e.g. during the winter period the maximum  $\delta$  of the Antarctic vortex is 1.8 compared with 3.0 for the Arctic vortex). Although these inter-hemispheric differences in the polar vortices are well known (e.g. Schoeberl and Hartmann 1991; Manney *et al.* 1995), the above diagnostics enable the differences to be



Figure 10. As in Fig. 6 except for  $N_2O$  in the southern hemisphere from 1 April to 31 December.

clearly shown and to be quantified. Another well-known characteristic of the southern winter which can be seen in Fig. 10 is the 'quiet' mid-winter period (Hirota *et al.* 1983; Randel 1988). The polar vortex is more distorted (i.e.  $\delta$  and  $\varphi_c$  are larger than the mean values) during April–May and September–November than during June–August.

At this stage the vortex orientation,  $\theta$ , has not been discussed in any detail. Figure 10(e) shows that there are several periods when there is a regular variation in  $\theta$ ; these correspond

to periods when there is regular rotation of a vortex. The vortex generally rotates eastward but there is considerable variation in the rotation rate. The eastward rotation of the vortex (with variable period) is consistent with observations of an eastward-travelling wave 2 in the southern hemisphere (e.g. Shiotani *et al.* 1990; Manney *et al.* 1991). Note that the rotation period of the vortex is twice the time for a major axis (ridge) to pass over a fixed location.

During the period when there is a strong vortex (May–November)  $\epsilon$  is very small, implying that the Antarctic vortex can be well approximated by an elliptical vortex. This together with  $\delta < 1.7$  suggests that the large-scale breaking events that occurred in the northern hemisphere (e.g. Figs. 1 and 7) did not occur in the southern hemisphere. If a higher value of N<sub>2</sub>O is used to define the vortex there are larger variations of  $\epsilon$  and  $\delta$ , but they are still smaller than for the Arctic vortex. Examination of N<sub>2</sub>O maps confirms that the Antarctic vortex is less disturbed than the Arctic vortex.

## 5. VERTICAL STRUCTURE

In the previous section the elliptical diagnostics were used to analyse the temporal evolution of the polar vortices on a single isentropic surface. In this section the vertical structure of the vortices is examined by comparing the diagnostics calculated on different isentropic surfaces.

First the 21 day period considered in section 3 is examined. The diagnostics are calculated for N<sub>2</sub>O on isentropic surfaces at 50 K intervals between 450 K and 700 K. Figure 11 shows the equivalent ellipse of a contour in the vortex-edge region at each level, at the beginning (18 December), middle (28 December) and end (7 January) of this period. Initially the vortex tilts equatorward and westward with height and is nearly circular at each level. In the middle of the period (during the wave breaking event, see section 3) the vortex still tilts westward with height but there is now only a very small meridional tilt. Also, the elongation of the vortex increases during this time, and the vortex is more elongated at upper levels than at lower levels. At the end of the period the vortex has very little vertical structure, and is a vertically aligned elliptical column centred off the pole. Note that in the 'stacked plots', Figs. 11(a)-(c), the apparent elongation of the vortex depends on the orientation of the vortex relative to the viewing direction: the elongation is under-emphasized if viewed along the semi-major axis, and exaggerated if viewed along the semi-minor axis. Also, as the area of the ellipse (vortex) at each level is sensitive to which contour is used to define the vortex (e.g. Fig. 3; see also Norton and Carver 1994 and Nash et al. 1996), it is difficult to say anything conclusive about the variation of vortex area with height from these plots or the elliptical diagnostics. Figure 12 shows the temporal variation of  $\varphi_c$ ,  $\lambda_c$ ,  $\delta$ , and  $\theta$  on each isentropic surface during the above period. These plots clearly show that the vertical structure is rapidly changing during this period. There is a large meridional tilt with height over the first half of the period, but very little tilt at the end (Fig. 12(a)). There are also strong variations in the east-west tilt of the vortex (Fig. 12(b)), with the tilt varying between westward (e.g. 20-23 December, 27-31 December) and eastward (2-4 December) with height. At the beginning and end of the period the vortex is nearly circular at all levels (Fig. 12(c)), but during the breaking event in the middle of the period there are strong variations in the aspect ratio (with  $\delta$  decreasing at lower levels first). Also, during this time of maximum elongation there is a strong westward tilt in the axis of elongation (Fig. 12(d)). The elliptical diaganostics have been calculated on the above isentropic surfaces for the complete cycle of both the Antarctic and Arctic vortices. Figure 13 shows the evolution of  $\varphi_c$ ,  $\lambda_c$  and  $\delta$  for 'edge' contours at the bottom (450 K; solid curves) and top (700 K;



Figure 11. Elliptical fit to the vortex edge at 450 K, 500 K, ..., 700 K isentropic surfaces at (a) and (d) 18 December, (b) and (e) 28 December, and (c) and (f) 7 January. Figures (a)–(c) show the vortex from a side view, whereas Figs. (d)–(f) are from the top view. Circles mark the pole and asterisks the vortex centre at each level. The values of N<sub>2</sub>O used to define the vortex edge are 210, 180, 140, 110, 70, 60 p.p.b. at 450, 500, ..., 700 K respectively.

dashed curves) surfaces for both vortices. These plots show many interesting features about the vertical structure of the polar vortices, only a few of which are mentioned briefly; a more detailed analysis is left for future studies.

First consider the Antarctic vortex (Figs. 13(a)-(c)). There is a reasonable correlation between  $\varphi_c$  at the different altitudes, implying that when the Antarctic vortex moves off the pole it does so throughout the lower and middle stratosphere. Note that this correlation breaks down during the vortex breakdown in October and November (not shown). The variation of  $\lambda_c$  with altitude is generally larger than that of  $\varphi_c$ , and there is a noticeable increase in the difference in  $\lambda_c$  (east-west tilt of the vortex) in late winter (September). The east-west tilt varies throughout the year, but there is a predominance for westward tilt with height. There are also strong variations of  $\delta$  with altitude although, as with  $\varphi_c$ , there is generally a good correlation between  $\delta$  at different altitudes. During early winter (May-June) the vortex is generally more elongated at lower altitudes, whereas in late winter (September-October) the opposite is true. The Arctic vortex (Figs. 13(d)-(f)) has many of the features of the Antarctic vortex: reasonable correlation between  $\varphi_c$  at the two levels, predominance for westward tilt with height, and generally more elongation at lower altitudes. However, the variations of location and shape with altitude are significantly larger than those of the Antarctic vortex. At several stages the centre of the Arctic vortex at 700 K is over 10° equatorward of the centre at 450 K (e.g. late-January, mid- and late-April), the east-west tilt of the Arctic vortex is generally larger than the Antarctic vortex, and the differences in aspect ratio of the Arctic vortex at the two different levels are often very large (e.g. in mid-January and early-March  $\delta \approx 1.6$  at 700 K while  $\delta \approx 2.2$  at 450 K).



Temporal variation of (a)  $\varphi_c$ , (b)  $\lambda_c$ , (c)  $\delta$ , and (d)  $\theta$  on the 450 K (solid curve), 500 K (dotted), 550 K Figure 12. (short dashed), 600 K (dash-dot) 650 K (dash-dot-dot), and 700 K (long dashed) isentropic surfaces for the period shown in Figs. 1 and 9.

An interesting feature of  $\varphi_c$  and  $\delta$  for both vortices is that there are many periods when the extremum occurs at 450 K, a day or so before that at 700 K, e.g. late June and early September in Fig. 13(a). This indicates that during these periods the disturbance (which causes the movement off the pole or elongation of the vortex) is propagating up the vortex. Further examination of this 'time lag' (e.g. Randel 1987) should provide information on the upward propagation of disturbances to the vortex.

#### 6. **COMPARISON WITH FOURIER DECOMPOSITION**

In this section the elliptical diagnostics are compared with the diagnostics from a Fourier decomposition of the zonal structure in the N<sub>2</sub>O data. In particular the possibility of extracting information about their location and elongation of the vortices from the Fourier analysis is examined. Some people may be tempted to infer changes in the location of the vortex from changes in the amplitude  $A_1$  and phase  $\Theta_1$  of zonal wave number 1 (wave 1), and to infer changes in the aspect ratio and orientation of the vortex from changes in the amplitude  $A_2$  and phase  $\Theta_2$  of zonal wave number 2 (wave 2). However, in the appendix idealized planar vortices are considered, with isolines that are ellipses with the same centre, aspect ratio and orientation. It is shown that for these vortices it is not always possible to infer changes in their location or elongation from the results of the Fourier analysis.



Figure 13. Temporal variation of  $\varphi_c$ ,  $\lambda_c$ , and  $\delta$  (see text) for N<sub>2</sub>O = 60 p.p.b. contour on 700 K (dashed curve) and N<sub>2</sub>O = 200 p.p.b. contour on 450 K surface (solid curve) in the southern (a)–(c) and northern (d)–(f) hemisphere.



Figure 14. Temporal variation of (a) A<sub>1</sub> and (b) A<sub>2</sub> at 61.5°S from 1 May to 30 November, and of (c) A<sub>1</sub> and (d) A<sub>2</sub> at 61.5°N from 1 October to 30 April. The dot-dashed curves correspond to (a) φ<sub>c</sub> from Fig. 8, (b) δ from Fig. 8, (c) φ<sub>c</sub> from Fig. 6, (d) δ from Fig. 6. (See text for further explanation.)

In particular, changes in  $A_1$  are not necessarily due to changes in  $\varphi_c$  (as these changes could be due to changes in  $\delta$  or the radial profile of the vortex), and changes in  $A_2$  are not necessarily due to changes in  $\delta$ .

To examine this further the amplitude and phase of waves 1 and 2 (at latitude 61.5°) have been computed for the 700 K N<sub>2</sub>O data examined in section 4. Figure 14 shows the temporal variation of  $A_1$  and  $A_2$  for the southern and northern hemispheres; also shown are the variations of  $\varphi_c$  and  $\delta$  (these are the same curves as in Figs. 6 and 10).

First the southern hemisphere is considered. From Figs. 14(a) and (b) it is seen that there is a good correlation between  $A_1$  and  $\varphi_c$  (linear correlation coefficient = 0.83), and between  $A_2$  and  $\delta$  (correlation coefficient = 0.88). This suggests that the amplitude of waves 1 and 2 may be used to infer the movement of the Antarctic vortex off the pole and the elongation of the vortex. However, the peak values of  $A_1$  are noticeably larger in May–June than in October–November, and if  $A_1$  were to be used to infer the latitude of the vortex centre it would be concluded that the vortex was further from the pole in May–June than in October–November. The time series of  $\varphi_c$ , and visual inspection of N<sub>2</sub>O maps, show that this is not the case.

The correlation between the wave amplitudes and the elliptical diagnostics in the northern hemisphere is significantly poorer than for the southern hemisphere (with the correlation coefficients less than 0.5), see Figs. 14(c) and (d). Although the extrema of  $A_1$  and  $\varphi_c$  generally occur at the same time (and there is reasonable correlation on the monthly

time-scale) there is a seasonal trend in  $A_1$  that is not present in  $\varphi_c$ ;  $A_1$  is much larger in December than in April, but  $\varphi_c$  is roughly the same for these two periods. As noted earlier, this is also seen to a lesser degree in the southern hemisphere. The analysis in the appendix, together with the evolution of the vortex area shown in Fig. 6, suggests that this trend in  $A_1$  may be due to the seasonal evolution of the meridional gradients of the vortex: the meridional distribution of N<sub>2</sub>O changes dramatically during the annual cycle (the vortex in April is smaller and has a sharper edge than in December) and these changes will cause a change in the wave amplitudes even if the vortex centre and elongation are unchanged. So the differences between  $A_1$  in December and April do not necessarily imply changes in the location of the vortex.

The difference between the evolution of  $A_2$  and  $\delta$  in the northern hemisphere is even larger than between  $A_1$  and  $\varphi_c$ . There are many periods when  $\delta > 2$  but  $A_2$  is small (and when  $A_2$  is large but  $\delta$  is small). Some of this difference could also be explained by the seasonal evolution of the meridional gradients. However, there are large differences on the weekly time-scale which suggest that this is not the only reason. For example, in early and late February there are, as discussed in section 3, wave breaking events in which the vortex becomes very elongated, but only the late-February event shows up in  $A_2$ . During this period the vortex has steep edge-gradients, is centred off the pole, and is elongated. Further examination of the idealized vortices considered in the appendix shows that, under these circumstances, an increase in the aspect ratio of the vortex does not necessarily result in an increase of  $A_2$ .

Finally, it is worth noting that if the wave amplitudes were used to infer the location and elongation of each polar vortex, the wrong conclusions about inter-hemispheric differences would be reached. Figure 14 would then imply that (i) the movement of the vortex off the pole in extreme events in the southern hemisphere is larger than in the northern hemisphere (as peak values of  $A_1$  are larger in the southern hemisphere) and (ii) on average the Antarctic vortex is more elongated than the Arctic vortex (as the average  $A_2$  is larger in the southern hemisphere). But in both cases the elliptical diagnostics (Figs. 6 and 10), and visual inspection of N<sub>2</sub>O maps, show that the opposite is true.

The above comparison suggests that, although the wave amplitudes from a Fourier decomposition of the data may provide some qualitative information about the location and elongation of the vortex, it is very difficult to extract quantitative information of these vortex characteristics.

# 7. CONCLUDING REMARKS AND FUTURE WORK

In this paper a set of diagnostics, based on the spatial moments of isentropic isopleths of quasi-conservative tracers (e.g. PV or a long-lived tracer such as  $N_2O$ ), has been presented that enables the structure of stratospheric polar vortices to be concisely summarized and quantified. These 'elliptical' diagnostics define the area, centre, aspect ratio and orientation of the polar vortex, and hence enable the movement and elongation of the vortex to be quantified. Although the area diagnostic is sensitive to the contour used to define the vortex (see also Norton and Carver 1994 and Nash *et al.* 1996), the other diagnostics are not for contours within the vortex-edge region. By examining the variations of these elliptical diagnostics with time and altitude, the temporal evolution and vertical structure of the polar vortices can be quantified. The diagnostics also enable the occurrence of large-scale wave breaking events to be identified, not only at the edge of the vortex but also at the edge of the tropics.

The usefulness of these diagnostics was assessed by using them to examine the stratospheric polar vortices (as defined by isopleths of  $N_2O$ ) in the SKYHI model. The temporal

variation of the diagnostics characterized the evolution of the vortices as shown by maps of N<sub>2</sub>O, and showed several differences between the Arctic and Antarctic vortices. The Arctic vortex moves further off the pole (at 700 K the mean latitude of the Arctic vortex is 85°N compared with 81°S for the Antarctic vortex), and is generally more elongated than the Antarctic vortex (at 700 K the maximum aspect ratio for the Arctic vortex is 3:1 compared with 1.8:1 for the Antarctic vortex). Also, the Arctic vortex has a more complicated vertical structure than the Antarctic vortex, with larger variations in both the vortex centre and elongation with height.

Comparison of the elliptical diagnostics with the results of a Fourier decomposition of the zonal wave structure has shown that, while the amplitudes of waves 1 and 2 provide qualitative information on the movement and elongation of the polar vortices, it is difficult to extract quantitative information from the wave amplitudes. This is because, even if the vortex location and elongation are fixed, the wave amplitudes will change if the meridional tracer gradients change. Also, in general, the amplitude of wave 1 depends on the elongation of the vortex as well as the location of the vortex centre (and similarly for the amplitude of wave 2).

The goal of this study was to assess the usefulness of spatial moments of tracer isopleths as diagnostics of stratospheric polar vortices. Having shown that these diagnostics can provide valuable information there are several possible applications.

Currently these diagnostics are being applied to several years of analysed PV. This will enable a climatology of the observed stratospheric polar vortices to be compiled, and for the interannual and seasonal variability of the vortices to be examined. The resulting diagnostics will also enable an examination of the vertical propagation of disturbances, either by a time-lag correlation analysis (e.g. Randel 1987) or by a detailed examination of individual events.

The diagnostics could also be used with satellite measurements of chemical tracers (e.g. measurements from the Upper Atmosphere Research Satellite, or the Total Ozone Mapping Spectrometer) to examine the structure of the polar vortices and of the Antarctic ozone hole. It will be interesting to compare the elliptical diagnostics calculated from chemical tracers and from analysed PV; the differences may show the effects of chemical or diabatic processes.

Finally, the elliptical diagnostics should enable closer comparison between the observed polar vortices and models; not only GCMs but also idealized models. For example, the elliptical diagnostics will allow direct comparison with the analytical solution for a twodimensional elliptical vortex in a uniform strain field (e.g. Kida 1981), and with contour dynamics simulations of topographically forced vortices (e.g. Polvani and Plumb 1992; Dritschel and Saravanan 1994).

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#### APPENDIX

To examine the relationship between the elliptical diagnostics and conventional Fourier analysis we consider a family of idealized vortices in which the isolines of a field f are ellipses of common origin ( $r_c$ ,  $\phi_c$ ), aspect ratio ( $\delta$ ), and orientation ( $\phi_0$ ). We compare the results of the Fourier analysis with the elliptical parameters. In particular, we compare the amplitude  $A_1$  and phase  $\Theta_1$  of waves with azimuthal wave number 1 (wave 1) with  $r_c$  and  $\phi_c$ , and the amplitude  $A_2$  and phase  $\Theta_2$  of wave 2 with  $\delta$  and  $\phi_0$ .

The distribution of f is given by

$$f(r, \phi) = \{1 - g(r, \phi)\}^n,$$
 (A.1)

where

$$g(r,\phi) = \{r\cos(\phi - \phi_0) - r_c\cos\widehat{\phi}_c\}^2 + \delta^2 \{r\sin(\phi - \phi_0) - r_c\sin\widehat{\phi}_c\}^2, \quad (A.2)$$

 $(r, \phi)$  are polar co-ordinates,  $\hat{\phi}_c = \phi_c - \phi_0$ , and *n* is a positive integer which determines the different members of the family. Varying *n* varies the decay of *f* from the vortex centre  $(r_c, \phi_c)$  but not the shape of the isolines, i.e. the radial distribution is given by

$$f(\widehat{r}) = (1 - \widehat{r}^2)^n,$$

where  $\hat{r}$  is the equivalent radius from the centre of each elliptical isoline ( $\hat{r} = \sqrt{A/\pi}$  where A is the area of the ellipse). As n increases, the vortex becomes more compact and has larger gradients at its 'edge', see Fig. A.1. Note we only consider here the analysis for  $\hat{r} < 1$ , so that the field f is always monotonically decreasing from the centre of the vortex.

The function g given by (A.2) can be rewritten as a Fourier series (involving only wave numbers less than 3):

$$g(r,\phi) = \frac{1}{2}a_0 + a_1\cos\phi + b_1\sin\phi + a_2\cos(2\phi) + b_2\sin(2\phi), \qquad (A.3)$$



Figure A.1. Meridional profile of the vortices f(r) for n = 1, 2, 4 and 8. See text for further explanation.



Figure A.2. Variation of amplitude of (a) wave 1 ( $A_1$ ) and (b) wave 2 ( $A_2$ ) with radius of vortex centre ( $r_c$ ) for circular vortex with distribution given by (A.1) with n = 1, 2, 4, 8.

where

$$\begin{aligned} a_0 &= 2r_{\perp}^2 (1+\delta^2) + r_{\rm c}^2 (\cos^2 \hat{\phi}_{\rm c} + \delta^2 \sin^2 \hat{\phi}_{\rm c}), \\ a_1 &= -2rr_{\rm c} (\cos \hat{\phi}_{\rm c} \cos \phi_0 - \delta^2 \sin \hat{\phi}_{\rm c} \sin \phi_0), \\ b_1 &= -2rr_{\rm c} (\cos \hat{\phi}_{\rm c} \sin \phi_0 + \delta^2 \sin \hat{\phi}_{\rm c} \cos \phi_0), \\ a_2 &= \frac{1}{2}r^2 \cos(2\phi_0)(1-\delta^2), \\ b_2 &= \frac{1}{2}r^2 \sin(2\phi_0)(1-\delta^2). \end{aligned}$$

We first consider the case n = 1. From (A.1) and (A.3) it follows that the amplitude and phase of waves 1 and 2 are:

$$A_{1} = 2rr_{c}(\cos^{2}\widehat{\phi}_{c} + \delta^{4}\sin^{2}\widehat{\phi}_{c})^{1/2},$$
  

$$\tan \Theta_{1} = (\cos \widehat{\phi}_{c} \sin \phi_{0} + \delta^{2} \sin \widehat{\phi}_{c} \cos \phi_{0})/(\cos \widehat{\phi}_{c} \cos \phi_{0} - \delta^{2} \sin \widehat{\phi}_{c} \sin \phi_{0}),$$
  

$$A_{2} = \frac{1}{2}r^{2}(\delta^{2} - 1),$$
  

$$\tan \Theta_{2} = \tan(2\phi_{0}).$$

Hence for vortices of this form there is a one-to-one relationship between wave 2 amplitude  $A_2$  (phase  $\Theta_2$ ), at given radius r, and the aspect ratio  $\delta$  (orientation  $\phi_0$ ) of the vortex, i.e. the values of  $A_2$  and  $\Theta_2$  do not depend on the location of the vortex centre. However, there

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is not a one-to-one relationship between  $A_1$  ( $\Theta_1$ ) and  $r_c$  ( $\phi_c$ ). For example, two vortices with the same centre  $(r_c, \phi_c)$  but with differing aspect ratio  $\delta$  will have different  $A_1$  and  $\Theta_1$ (at the same r).

The n = 1 family is in fact a special case. For n > 1 there is no longer a one-to-one relationship between  $\delta(\phi_0)$  and  $A_2(\Theta_2)$ ; in general  $A_2$  will depend on the vortex centre as well as its aspect ratio. This means that, in general, changes in amplitude of wave 1 (wave 2) do not necessarily imply changes to the centre (elongation) of the vortex.

To illustrate this, Fig. A.2 shows the variation of  $A_1$  and  $A_2$  (evaluated at r = 0.5) with the location of vortex centre for a circular vortex with radial distribution given by n = 1, 2, 4, and 8. As shown above,  $A_1$  varies linearly with  $r_c$ , and  $A_2 = 0$  when n = 1. However, for larger n,  $A_2$  increases from zero as the vortex moves away from the origin (with A<sub>2</sub> increasing more rapidly for more localized vortices), and there is no longer a linear nor a monotonic relationship between  $A_1$  and  $r_c$ . Furthermore,  $A_1$  and  $A_2$  will change for a circular vortex with fixed centre if the meridional profile of the vortex (i.e. n) changes; that is to say, the amplitude of waves 1 and 2 can vary even if the centre and shape of the isolines do not change. For vortices with non-circular isolines the connections between  $A_1$ and  $A_2$  and  $r_c$ ,  $\phi_c$ ,  $\delta$  and  $\phi_0$  are even more complicated than shown in Fig. A.2.

Hence for the above vortex distributions there is not, in general, a simple relationship between the elliptical diagnostics and the results from a Fourier analysis of the distribution.

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