1 Particle dark matter

We discuss some issues surrounding neutrinos as potential sources of dark matter. Although we will reject this hypothesis, neutrinos represent a prototype for other possible sources of particle dark matter, generically known as WIMPs. We will then discuss some possibilities for WIMPs and finish with some the difficult task of detecting WIMPs.

1.1 Neutrino decoupling

Figure 1 was also shown earlier in an earlier lecture. In this case we highlight the point at which neutrino decoupling occurs. Figure 2 gives another history of the universe, showing the main quantities of interest as powers of ten. At times prior to decoupling, the neutrinos are in equilibrium with the rest of the particle and radiation of the expanding universe according to equilibrium reactions such as

\[ \gamma \leftrightarrow e^+ e^- \leftrightarrow Z^0 \leftrightarrow \nu \bar{\nu} , \]

where \( \gamma \) is a photon and the \( Z^0 \) particle is the neutral carrier of the weak interaction (see the particle physics primer). The large mass of the \( Z^0 \) (about 91 GeV/c\(^2\)) accounts for the short range of the weak interaction. Since neutrinos only couple to the weak force, they cannot directly interact with photons. As long as the neutrinos are frequently exchanging energy in reactions like this, then they will remain in thermal equilibrium. As the universe expands and cools, at some point the particles will no longer stay in thermal contact – this is decoupling. We encountered this earlier in the discussion of the decoupling of photons, thus giving rise to the cosmic microwave background. We now try to determine the point of decoupling.

We can use a simple relation to understand the point where decoupling occurs. As discussed previously, the particle collision rate is given by \( \Gamma = n \sigma v \), where \( n \) is the particle density of the interacting particles, \( \sigma \) is the interaction cross section, and \( v \) is the relative speed of the colliding particles. We simply compare this collision rate with the rate of expansion of the universe, which is \( H = \dot{R}/R \). So within factors of order 1, we find:

\[ \Gamma = n \sigma v > H \Rightarrow \text{equilibrium} ; \quad (1) \]

\[ \Gamma = n \sigma v < H \Rightarrow \text{decoupling} . \quad (2) \]
We can now input the known physics to determine the factors above. When we do, we find that $\Gamma = H$ at $kT \approx 1 \text{ MeV}$, when the age of the universe was approximately 1 s.

1.2 Cosmic neutrino background

In class, we simply looked at the temperature dependence of each term of Eqs. 1-2 to determine that

$$\Gamma / H \propto T^3.$$  \hspace{1cm} (3)

So the temperature of decoupling varies rapidly with collision rate, for example. We can use these ideas to qualitatively see how neutrino decoupling influenced formation of light nuclei, which began occurring slightly later, when $t \approx 1 \text{ hr}$. Since we saw earlier that $H \propto g_*^{1/2} T^2$, where $g_*$ increases with the number of available particle species, then a larger number of neutrino types, $N_\nu$, increases $H$ and therefore, by Eq. 3, the decoupling temperature in-
creases (decoupling occurs earlier). This in turn means that the equilibrium ratio of neutron to proton density,
\[ \frac{n}{p} = e^{-Q/kT}, \]
where \( Q = (m_n - m_p)c^2 = 1.3 \text{ MeV} \), is larger at decoupling. The larger \( n/p \) ratio then implies a larger fraction of neutron-rich nuclei. In particular, a larger \( N_\nu \) gives rise to a predicted \( ^4\text{He}/H \) ratio which is larger.

The quantitative versions of the above arguments were used in the 1980’s to predict the number of neutrino species in nature to be about \( N_\nu \leq 4 \) based on the observed nuclei ratios. Subsequently, in 1989, accelerator experiments were able to measure \( N_\nu \) accurately. The method used was to produce the \( Z^0 \) particle in the process \( e^+ + e^- \rightarrow Z^0 \). This produces a resonance when the collision energy is equal to the mass of the \( Z^0 \approx 91 \text{ GeV}/c^2 \), as shown in Fig. 3. The width of this quantum state depends on the number of possible
decay modes, and hence $N_\nu$. The current experimental result is

$$N_\nu = 3.00 \pm 0.01.$$ 

Hence the prediction based on cosmic astrophysics was confirmed.

Figure 3: Accelerator experiment result showing the excitation of the $Z^0$ resonance. The curves show the theoretical expectations for different assumptions for the number of neutrino species.

The neutrinos which decoupled from the cosmic fireball at $t \approx 1$ s, in analogy to the 2.73 K photon background, should still be around. As discussed in the previous lecture, the predictions are that the density of these neutrinos is

$$n_\nu = 113 \times N_\nu \approx 340 \text{ cm}^{-3},$$

with an average thermal temperature now of 1.95 K.

1.3 Neutrinos as dark matter

We saw in Lecture 7 that there is now good evidence that all 3 neutrino species have a small, but finite, rest mass, $m_\nu \sim 10^{-3}$ eV/c$^2$. But since there
are a lot of them around us from the decoupling, then perhaps this gives an appreciable contribution to the estimated dark matter mass density. We find that to make a critical density from cosmic neutrinos requires that the mass of the 3 species total $47 \, \text{eV/}c^2$, that is

$$\rho_\nu = \rho_c \Rightarrow \rho_\nu = 340 \times 47 \, \text{eV/cm}^3.$$  \hfill (4)

We would require about 30% of this in order for neutrinos to give the “right” contribution to dark matter. The data suggests it is about $10^3$ short of this. So it seems that neutrinos contribute rather insignificantly to dark matter.

There is one other reason why it was not expected that neutrinos would contribute to this. Structure formation (companion lecture) demands that the dark matter be “cold,” that is, that the particles be non-relativistic during the early era of structure formation. Recall that the neutrinos decouple with an average energy $kT \approx 1 \, \text{MeV}$, much larger than the rest energies. Indeed, at the dawn of structure formation, the neutrinos would have only cooled to $\sim 10 \, \text{eV}$, still relativistic. Therefore, neutrinos are not only too light, but also too “hot” to be an important contributor to dark matter.

### 1.4 WIMPs

We didn’t discuss this in much depth in class. But this is an important topic for both astrophysics/cosmology and elementary particle physics (aka HEP). An important issue in HEP theory is that basic calculations become untenable at energies which are about 10 times higher than the reach of our current accelerators, which currently probe elementary particle interactions at energies of $\sim 200 \, \text{GeV}$. One solution to this is a theory called *supersymmetry*, aka SUSY. A prediction of SUSY is that there is a stable, neutral, weakly interacting particle, usually called the neutralino, $\chi^0$. For our present purpose we can simply think of these as heavy neutrinos. Generically, such dark matter candidates are called WIMPs, for weakly-interacting massive particle.

Just as with neutrinos, we can evaluate WIMP decoupling using Eqs. 1 and 1. However, because they are massive, they will be non-relativistic at decoupling, and hence will contribute to cold dark matter (CDM). The decoupling temperature is about $Mc^2/kT \approx 25 \pm 5$ for a WIMP of mass $M$. The WIMP energy distribution will follow a Boltzmann function after decoupling. Generically, the WIMP energy density is found to follow

$$\rho_{\text{wimp}}/\rho_c \sim \left[10^{-25} \, \text{cm}^3\text{s}^{-1}\right]/(\sigma v).$$
Figure 4 gives this critical density as a function of WIMP mass $M$, basically reflecting the denominator in the equation above. Qualitatively, we see three branches in this figure. The first, at low mass, corresponds to light, relativistic WIMPs. Neutrinos fall on this branch, but off scale at low mass. The next branch, at intermediate mass, is for WIMPs which are non-relativistic at decoupling. For these masses, the weak cross section varies as $\sigma \propto G_F^2 M^2$. The form of the weak cross section changes for masses near the mass of the weak force quanta, the $W$ and $Z^\circ$ particles, which have masses of about 90 GeV/c$^2$, to

$$\sigma \propto G_F^2 \left[ \frac{M^2}{(M^2 + M_Z^2)^2} \right],$$

which gives the third branch. The shaded region is the range of cosmological interest for dark matter. This third branch is the one where one would expect the SUSY particle.

### 1.5 WIMP detection

The best and clearest way to test the WIMP particle dark matter solution is to look at data from the next generation of particle accelerator, such as the LHC in Europe, and the proposed linear collider. These accelerators will probe exactly the energy range given by the intersection of the third branch and the shaded region in Fig. 4. The SUSY version of this solution would be that the WIMP be the neutralino, $\chi^\circ$. In the accelerator interactions, one could test if the observed WIMP had the right properties, including stability against decay, to be a real manifestation of cold dark matter.

On the other hand, one would also like to directly observe the relic WIMPs left over from cosmic decoupling. In an earlier lecture, we showed the difficulty of measuring neutrino interactions in accelerators. WIMP detection is even more challenging, since the kinetic energy, and hence the interaction rate with matter, is much, much smaller. A generic estimate for the rate ($R$) per unit detector mass for WIMP interactions in a detector utilizing a medium of atomic mass $A$ is roughly

$$R \sim 10^4/(AM) \text{ events/kg/day},$$

for a WIMP of mass $M$. This is experimentally very challenging.
Figure 4: $\Omega_w = \rho_{\text{wimp}}/\rho_c$ vs WIMP mass (eV). From Perkins