Thermodynamics & Statistical Mechanics PSet 13

1. **Book Problems:** 7.75, 8.23c, 5.32, 5.33, 5.81
2. **Book Problem Extra Credit:** 8.29, 8.32, 5.46
3. **Further Reading:** We may not quite finish the book, so I'd recommend reading some stuff we skipped in chapter 5 and in the last chapter. Relatedly, you might enjoy the phase transition section of these notes (http://www.damtp.cam.ac.uk/user/tong/statphys.html).

Study Guide

If you can explain all of these ideas to your fellow students without looking at your notes... especially if at the same time you can re-derive much of the associated math – then you're in really great shape. What's below isn't official homework, but I'd recommend trying to force yourself to explain the answers to these questions at the blackboard, and (perhaps) discuss/debate the best way to think about them with your friends.

1. Considering a large system at fixed energy, can you explain the relationship between energy U, multiplicity Ω (and the difference between micro and macro states), entropy S, and (finally) the temperature T? What really is the temperature? Why would you expect that the entropy can only ever stay the same or increase?
2. Why do a lot of independent random events (eg flipping a lot of coins and looking at total number of heads), when combined, have a Gaussian distribution? How large are fluctuations around the mean? How does this relate to the idea of the thermodynamic limit?
3. How does energy conservation (very easy) and the 2nd law of thermodynamics constrain the efficiency of engines? Make sure you can calculate the efficiency of an engine directly, at least if you know its working fluid is an ideal gas.
4. If we work at constant temperature, so that our system is in contact with a reservoir of energy – how do you derive Boltzmann factors? What's the probability of a system being in a given microstate, and how does that relate to the partition function Z? How can you calculate the energy, entropy, and specific heat from Z?
5. Why are thermodynamic potentials useful? What are the thermodynamic potentials F, H, G – what quantities (eg V, P, T, S, N, μ) are they functions of? How can we relate F to Z? How do we choose which thermodynamic potential to use in a given situation?
6. To do quantum statistical mechanics we re-oriented our description of states and switched to using the Gibbs ensemble. What exactly did we do, and why did we do this? When do we need quantum statistical mechanics and when can we rely on classical statistical mechanics? What are the Fermi-Dirac and Bose-Einstein distributions?

7. How does a gas of free fermions behave? What is the Fermi energy and why is it important? How does the specific heat of a free Fermi gas depend on temperature, and how can you explain this scaling intuitively?
8. What's the energy density of a gas of free photons... how does it scale with temperature, and why? How does this relate to black body radiation, and what's the Stefan-Boltzmann law for the emission of radiation? What's the Debye temperature of a solid?
9. What's Bose-Einstein condensation, and why does it occur? It's a trivial fact that at very low temperatures any system will be in its ground state, so why is BEC interesting?
10. What's a phase transition, and how/why can we use the Gibbs free energy to study them? What's a critical point?
11. What's the entropy of a probability distribution? What's the relationship between entropy and the amount of information that you can communicate?