

## Thermodynamics & Statistical Mechanics PSet 12

1. **Book Problems:** 7.51, 7.52, 7.54, 7.64, 7.66, 7.72, 7.73
2. **Natural Units (Extra Credit):** The goal is to write some important quantities in ‘natural’ units, so we can better understand how material properties follow from fundamental physics. First show that if we work in units where  $\hbar = c = 1$  (also  $k_B = 1$ ) then temperature, energy, and momentum have the same units, while distance and time have units of 1/energy. So for example the electron mass  $m_e$  picks out a corresponding length scale  $1/m_e$ . Also, show that the fine structure constant  $\alpha \equiv \frac{e^2}{4\pi\epsilon_0\hbar c}$  is dimensionless, so it’s of the order of  $10^{-2}$ , so it’s meaningfully small. (This is a dimensionless sense in which the electromagnetic force between fundamental charges is weak.)

With that established, write the Rydberg energy and Bohr radius in terms of  $m_e$  and  $\alpha$ . Make a natural, rough guess for the average separation between atoms in a solid in terms of  $m_e$  and  $\alpha$ , and then use this to make a rough estimate of the Fermi energy  $\epsilon_F$ .

Now try to make a guesstimate of the Debye energy  $T_D$  in terms of  $\alpha$ ,  $m_e$ , and the atomic mass  $M$ . How do you expect  $T_D$  to compare to  $T_F$  for a generic material?

If you liked this problem, for fun you can try to re-write everything else you’ve ever studied in this way in order to try to better understand why some numbers are large and some are small. For example, determine the ‘natural units’ of the Newton constant  $G_N$  and compare its magnitude to some power of  $m_e$ . How does the magnitude of the gravitational force between two electrons compare to the electric force? Why are stars so big? You may have learned about white dwarves and neutron stars from other recent problems... it should be possible to write their size in terms of  $G_N$  and other fundamental parameters.

3. **Book Problem Extra Credit:** 7.50, 7.53, 7.70