

# QFT Problem Set 4 - Due Oct. 27

You should read chapters 8-10 of the book. As usual, \*problems\* are for theorists and extra credit seekers, although everyone should look at them.

1. **Book Problems** 8.3, 8.5, 9.1, \*8.7\*, \*9.3\*,
2. **Integrating Out** Consider a theory with Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\chi^2 - \frac{1}{2}M^2\Phi^2 - \frac{g}{2}\chi^2\Phi \quad (1)$$

where we are interested in the limit that  $M \gg m$  and also  $M \gg E_{exp}$ , the energies at which we are conducting experiments (for example experiments on  $\chi$  particle scattering).

First note the Feynman rules for this theory. Compute the equation of motion for the  $\Phi$  field and write the solution. Next evaluate the Lagrangian on the solution to the  $\Phi$  equations of motion, and perform an expansion in  $1/M^2$ . Given the questions of interest, why is it justified to assume  $\square \ll M^2$ ? What kinds of interactions between  $\chi$  particles are generated to first and second order in  $1/M^2$ ? Work out the Feynman rules for this new Lagrangian, to second order in  $1/M^2$ . Finally, compute the scattering amplitude for 2-to-2 scattering of  $\chi$  particles to second order in  $1/M^2$ , both using the original Lagrangian and using the new Lagrangian, and check that the results match.

This procedure is called ‘integrating out’ a heavy field, and it is an example of the effective field theory philosophy, where we eliminated short distance physics and simply use an effective lagrangian for the long-distance physics (in this case,  $\chi$  scattering).

3. **LSZ in Action, or Field Redefinition Invariance** First consider the theory of a free massless complex scalar field

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi \quad (2)$$

Quantize this field (note that there are two kinds of creation and annihilation operators, which is easiest to understand if we write  $\phi$  using two real scalar fields and then recombine them to make the complex field).

Now write

$$\phi(x) = (v + \rho(x))e^{i\frac{\theta(x)}{v}} \quad (3)$$

where  $v$  is a real constant and  $\rho(x)$  and  $\theta(x)$  are both real scalar fields. Work out the Lagrangian in terms of  $\rho$  and  $\theta$ . You should see that it has non-trivial interactions, so quantize  $\rho$  and  $\theta$  and then work out the Feynman rules for their scattering. Finally, compute the scattering amplitude for 2-to-2 scattering of  $\theta$  particles using these rules, and show that it’s zero.

The lesson is that we can use whatever variables we want to represent the quantum fields – the S-Matrix will be the same, as long as our fields create one-particle states. The form of the Lagrangian doesn’t matter at all; it is not a physical observable.

4. **DIY Spontaneous Symmetry Breaking** Consider a theory with Lagrangian

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi - \mu^2 |\phi|^2 - \lambda |\phi|^4 \quad (4)$$

Solve the classical equations of motion for constant  $\phi = v$  when the parameter  $\mu^2 < 0$ . You should find that the re-phasing symmetry  $\phi \rightarrow e^{i\alpha} \phi$  is spontaneously broken, since you have to choose a phase for  $v$ . Choose  $v$  real and again write

$$\phi(x) = (v + \rho(x)) e^{i\frac{\theta(x)}{v}} \quad (5)$$

What is the spectrum (what are the particles and their masses)? Compute the leading 2-to-2 scattering amplitude for  $\theta$  particles in the limit that they have low energy. You can use the methodology from problem 1 if its easier.