QFT Problem Set 2 - Due Sept. 29

You should read chapters 3, 4, and 5 of the book. We'll be getting to 6 and 7 soon as well. As usual, *problems* are for theorists and extra credit seekers, although everyone should look at them.

1. **Shift Symmetries** What sort of Lagrangians have a symmetry under $\phi \to \phi + \alpha$? Write down a non-trivial (not just a free field theory) example and work out the conserved current. What are the implications for the mass of the ϕ particles? How does this relate to our theory of phonons?

2. **Book Problems** 3.1, 3.2, 3.5, 3.6, 5.2, *3.3*, *3.7*, *4.1*, *5.3*

3. **More Dimensional Analysis** Eventually we will study theories with fermions and photons, but for now it's enough to know that fermions have kinetic terms of the form

$$S \sim \int d^d x \, \psi^\dagger(x) \partial \psi(x) \tag{1}$$

with only one derivative, and you already saw the kinetic term for a photon

$$S = \int d^d x \left(-\frac{1}{4} F_{\mu\nu}^2 \right) \tag{2}$$

What are the dimensions of the photon field A_μ? Is there any spacetime dimension d where the coupling κ of the interaction $\kappa (F_{\mu\nu})^3$ is dimensionless? Argue that photon self-interactions must be very small effects at long-distances.

What are the dimensions of the fermion field ψ? What are the dimensions of the couplings g_1 and g_2 from the interaction terms $g_1 \phi \psi^\dagger \psi$ and $g_2 \phi^2 \psi^\dagger \psi$, where ϕ is a scalar field? Find a spacetime dimension d where g_1 is dimensionless, and a d where g_2 is dimensionless. Consider $\lambda (\psi^\dagger \psi)^2$, find the dimensions of λ, and find a spacetime dimension where λ is dimensionless.

Finally, note that the gravitational field h, at the linearized level, has a kinetic term like

$$S \sim \int d^d x (\partial h_{\mu\nu})^2 \tag{3}$$

and it interacts with e.g. a scalar field ϕ like

$$S \sim \int d^d x \sqrt{G_N} h_{\mu\nu} \partial^\mu \phi \partial^\nu \phi \tag{4}$$

What does this say about the dimensions (units) of the Newton constant G_N in $d = 4$ spacetime dimensions? You should see that it agrees with your analysis of the units of G_N from the first problem set. As you saw there, G_N is associated with a very large energy scale, meaning that gravity is a very weak force, which is only of interest to us because it is universally attractive.