Assignment 11

171.303 Quantum Mechanics I

Due: December 11, 2018

For all problems, consider the simple harmonic oscillator Hamiltonian

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

where m and ω are some positive constants.

Problem 1

A particle of mass m is in a one-dimensional harmonic oscillator. The state is such that measurement of the energy yields $\hbar\omega/2$ or $3\hbar\omega/2$ with equal probabilities of $\frac{1}{2}$. At time $t = 0$, $\langle x \rangle = \sqrt{\hbar/2m\omega}$.

(a) Show that this uniquely determines the state of the particle, and find the state.

(b) Find $\langle x \rangle$ and $\langle p_x \rangle$ as functions of time. Verify that these satisfy Ehrenfest's theorem

$$\frac{d}{dt}\langle p_x \rangle = \langle -\frac{dV}{dx} \rangle$$
$$\frac{d}{dt}\langle x \rangle = \frac{\langle p_x \rangle}{m}.$$

Problem 2

A particle is in the ground state of the harmonic oscillator. Instantaneously, the frequency of the oscillator changes from ω to $\omega' = 3\omega$; the state of the particle does not change immediately. Calculate the probability that the particle will be measured in the ground state for the new potential.

Problem 3

Consider a coherent state

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

which satisfies $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$.

(a) Find $\langle x \rangle$ and $\langle p_x \rangle$.

(b) Find $\langle E \rangle$ and ΔE.

(c) Suppose we have a system initially in the coherent state $|\psi(0)\rangle = |\alpha\rangle$. Show that some time t later, $|\psi(t)\rangle$ is another coherent state, and find the eigenvalue of $\hat{a}$ for this state.