Assignment 7

171.303 Quantum Mechanics I

Due: November 6, 2018

Problem 1

Consider a system of 2 spin-$\frac{1}{2}$ particles. Our system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle|-\mathbf{z}\rangle + \frac{i}{\sqrt{2}}|-\mathbf{z}\rangle|+\mathbf{z}\rangle.$$

(a) Is this state entangled? Why or why not?

(b) Rewrite the state $|\psi\rangle$ in the basis of eigenstates of total angular momentum $|j, m\rangle$. What are $\langle J^2\rangle$ and $\langle J_z\rangle$?

(c) What is the probability of finding particle 1 in the state $|+\mathbf{z}\rangle$? If we first measure particle 2 to be in the state $|-\mathbf{z}\rangle$, what is the the probability then that we find particle 1 in the state $|+\mathbf{z}\rangle$?

Problem 2

(a) Show that for any positive integer n, $[\hat{x}^n, \hat{p}_x] = i\hbar n\hat{x}^{n-1}$ through induction.

(b) Let $F(x)$ be a function, and $F(\hat{x})$ be the operator defined via the Taylor series expansion:

$$F(\hat{x}) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n F}{dx^n}\bigg|_{x=0} \hat{x}^n.$$

Using your result from part (a), show that $[F(\hat{x}), \hat{p}_x] = i\hbar\frac{dF}{dx}(\hat{x})$ (where the right hand side is also defined by its Taylor series).

(c) For the 1-dimensional Hamiltonian $\hat{H} = \frac{\hat{p}_x^2}{2m} + V(\hat{x})$ show that $\frac{d\langle p_x\rangle}{dt} = \langle -\frac{dV}{dx}\rangle$ using your results from part (b).

Note: Together with $\frac{d\langle x\rangle}{dt} = \frac{1}{m}\langle p_x\rangle$ which can be proven in a similar way, this shows that the expectation values of position and momentum have similar dynamics as their corresponding classical observables. However, they are *not exactly* the same, as $\langle\frac{\partial V}{\partial x}\rangle$ is not in general equal to $\frac{\partial V}{\partial x}(\langle x\rangle)$. For more details see the discussion on page 200 of the text.