Assignment 2

171.303 Quantum Mechanics I

Due: September 18, 2018

Problem 1

Consider the sequence of SG-apparatuses with one track blocked (with the same notation as in problem 4 of assignment 1):

$$\boxed{\text{SG } +\hat{\mathbf{y}}} \rightarrow \boxed{\text{SG } -\hat{\mathbf{x}}} \rightarrow \boxed{\text{SG } -\hat{\mathbf{y}}}$$

(a) As described in the text, each of these apparatuses corresponds to a projection operator, and putting them in sequence corresponds to the product. Write down, in bra-ket notation, the operator $\hat{A}$ corresponding to this sequence of measurements. Make sure the order is correct. Your operator should transform an initial *ket* state into its final state.

(b) Using the expressions,

$$\begin{aligned} |\pm\mathbf{x}\rangle &= \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle \pm \frac{1}{\sqrt{2}}|-\mathbf{z}\rangle \\ |\pm\mathbf{y}\rangle &= \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle \pm \frac{i}{\sqrt{2}}|-\mathbf{z}\rangle, \end{aligned}$$

evaluate the relevant inner products from your expression in part (a), and rewrite $\hat{A}$ in simpler terms.

(c) Based on your result from part (b), what fraction of a beam of electrons in an initial state $|+\mathbf{y}\rangle$ will make it through the experiment with a final state $|-\mathbf{y}\rangle$?

Problem 2

It's often easier to work with matrix representations of operators for the purpose of explicit calculations. In this exercise you will work out a matrix representation of the operator $\hat{J}_y$ in the S_z-basis.

(a) Evaluate $\hat{J}_y|+\mathbf{z}\rangle$ by inserting the identity in the form $|+\mathbf{y}\rangle\langle+\mathbf{y}| + |-\mathbf{y}\rangle\langle-\mathbf{y}|$. Then, do the same for $\hat{J}_y|-\mathbf{z}\rangle$.

(b) The matrix elements of an operator are $\mathbb{A}_{ij} = \langle i|\hat{A}|j\rangle$, where $|i\rangle$ and $|j\rangle$ are basis vectors. Applying $\langle \pm\mathbf{z}|$ to the left hand side of your results in part (a), calculate the matrix elements of $\hat{J}_y$ in the S_z-basis. Check that $|\pm\mathbf{y}\rangle$ are indeed eigenvectors with the correct eigenvalues in this representation.

(c) Now, compute the matrix representation (in the S_z basis) of the operator

$$\hat{O} = \frac{\hbar}{2}(|+\mathbf{y}\rangle\langle+\mathbf{y}| - |-\mathbf{y}\rangle\langle-\mathbf{y}|).$$

Compare with part (b) and explain your findings.

Problem 3

Suppose we have an electron in the state

$$|\psi\rangle = N\big(-|+\mathbf{z}\rangle + (-1+2i)|-\mathbf{z}\rangle\big).$$

(a) Find the value of N so that $|\psi\rangle$ is normalized. Choose N to be real and positive.

(b) Calculate the probability that a measurement of $\hat{J}_z$ in this state yields a value of $\frac{\hbar}{2}$. Then calculate the expectation value $\langle J_z\rangle$.

(c) What is the uncertainty in the measurement ΔJ_z?