Problem 1 (20 points)
A large collection of spin-$\frac{1}{2}$ particles are all in the state $|\psi\rangle = \alpha |+z\rangle + \beta |-z\rangle$, where $\alpha$ and $\beta$ are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$.

(a) What is the expectation value $\langle S_z \rangle$ for the $+z$ component of spin angular momentum? (6 points)

(b) What is the expectation value $\langle S_x \rangle$ for the $+x$ component of spin angular momentum? (6 points)

(c) We now send this collection of particles through a Stern-Gerlach machine $SG_y$, with field gradient in the $+y$ direction. What fraction of the particles will be found with spin up? (8 points)

Problem 2 (25 points)
A spin-$j$ particle is in a superposition of the $S_z = mh$ and $S_z = (m+1)\hbar$ states, where $-j \leq m \leq j - 1$. Both states are found to have equal probability of being measured.

(a) Write the most general possible state $|\psi\rangle$ describing the particle satisfying the given information. (10 points)

(b) Suppose we measure $\langle S_z \rangle = 0$. Use this information to further constrain the state $|\psi\rangle$. (15 points)
Problem 3 (20 points)
A spin-1 particle is in the state with $S_n = +\hbar$, where $n = \cos \theta \mathbf{x} + \sin \theta \mathbf{y}$. Find the probability that, upon measuring angular momentum along the $x$ axis, we measure $S_x = \hbar$.

Problem 4 (35 points)
A spin-$\frac{1}{2}$ particle has magnetic moment $\mu = \gamma S$, where $\gamma$ is a positive constant. It enters a uniform magnetic field of strength $B_0$ in the $+y$ direction. At time $t = 0$ the particle is in the $| +z \rangle$ state.

(a) What is the Hamiltonian for this system? (5 points)

(b) What is/are the energy eigenvalue(s) and eigenstate(s)? (5 points)

(c) What is the state of the particle at a subsequent time $t$? (8 points)

(d) Find $\langle S_z \rangle$ as a function of time. (5 points)

(e) Find $\langle S_y \rangle$ as a function of time. (5 points)

(f) Can you explain your result in part (e)? (7 points)