1. (30 points) A spin-2 particle is prepared in the state $|2, +2\rangle$, where $\hat{S}^2|2, +2\rangle = 2 \cdot (2 + 1)\hbar^2|2, +2\rangle$ and $\hat{S}_z|2, +2\rangle = 2\hbar|2, +2\rangle$. What is the probability that, when measured, it has zero angular momentum projection in the x-direction, i.e., $S_x = 0$?

2. (35 points) Consider the Hamiltonian

$$\hat{H} = \frac{2A}{\hbar^2}(\hat{S}_x\hat{S}_y + \hat{S}_y\hat{S}_x)$$

(a) Are there states which are simultaneous eigenstates of $\hat{S}^2$ and $\hat{H}$?
(b) For a spin-1/2 particle, compute $|\psi(t)\rangle$ if $|\psi(0)\rangle = |+\vec{z}\rangle$.
(c) For a spin-1 particle, compute $|\psi(t)\rangle$ if $|\psi(0)\rangle = |1, +1\rangle$.

3. (35 points) A beam of spin-1/2, particles in the $|+\vec{z}\rangle$ state is sent through a modified Stern-Gerlach experiment oriented in the x-direction, and then through a normal S-G device oriented in the z-direction. In the modified device the beam is split into two paths and the $S_x = -\hbar/2$ path passes through a magnetic field for a time $T$ oriented along the axis of motion (the y-direction) before it is recombined with the $S_x = +\hbar/2$ path. What is the minimum time the particles must spend in the magnetic field such that all of the particles leave the final SGz with $S_z = -\hbar/2$?

You can parametrize the energy of the particle in the magnetic field as $\omega_0 \times S_y$.

Note: Assume the magnetic field, while only affecting part of the quantum state, leaves the state 'coherent'. The Hamiltonian is only non-trivial for one of the paths.