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The *Studies in Applied Economics* series is under the general direction of Prof. Steve H. Hanke, Founder and Co-Director of the Johns Hopkins Institute for Applied Economics, Global Health, and the Study of Business Enterprise (hanke@jhu.edu).

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Abstract

The production of good money seems to be out of reach for most countries. The aim of this paper is to examine how a country can attain monetary stability by granting legal tender to two freely tradable currencies circulating in parallel. Then we examine how such a system of parallel currencies could be used for any Member State of the Eurozone, with both the euro and a national currency accepted as legal tender, which we argue is a desirable monetary arrangement particularly but not only in times of crisis. The necessary condition for this parallel system to function properly is confidence in the good behaviour of the monetary authorities in charge of each currency. A fully floating exchange rate between the two would keep the issuers of the new local currency in check. This bottom-up solution based on currency choice could also be applied in countries aspiring to enter the Eurozone, instead of the top-down once and for all imposition of the euro as a single currency that has turned out to be very stringent and has shown institutional flaws during the recent Eurozone crisis of 2009 – 2013. Our scheme would have alleviated the plight of Greece and Cyprus. It could also ease the entry of the eight Member States still missing from the Eurozone.

Keywords: Parallel currency system, monetary competition, inverse Gresham law, Eurozone

1. Introduction: a bottom up system to achieve monetary stability

A mixture of overconfidence and incomplete institutional design have made the Eurozone suffer monetary instability and political tensions. As with previous monetary unions (see Bordo and Jonung, 2003) the euro was launched for an ultimate political reason (Buiter, 1999; Goodhart, 2003; Schwartz, 2004): to contribute to the general goal of "an ever closer union among the peoples of Europe", as proclaimed in the founding Treaty of Rome. This political objective was pursued principally by indirect economic means, in the manner of the builders of the European Union from its very inception. Based on the Euro barometer data, Roth and Jonung, (2019) shows that there has been a large majority of support to the euro since its inception, even in the years of the Eurozone crisis and in the countries which have suffered the most the consequences of the crisis; however, the EU governments and the euro institutions such as the European Central Bank lost support from 2008 to 2013, which reflects the impact of the poor management of the Eurozone crisis in economic and social discontent both in the core and peripheral Member States during and after the Eurozone crisis. According to that study, the euro is associated with low inflation and low unemployment, so that its support very much depends achieving macroeconomic stability. We here propose a monetary arrangement to achieve that goal, alternative to the command route taken by the EU particularly during and since the Eurozone crisis.

The original design of the euro has proved to be flawed (see Goodhart, 2014). It was designed to be *a stable rule-based currency* issued by an independent central bank charged with the difficult task of applying a single monetary policy to widely differing nations and regions. A recent empirical study by Castaneda and Schwartz (2017) shows that macroeconomic asymmetries within the Eurozone had nearly doubled from 1999 to 2007 before the outbreak of the Global Financial Crisis. The dispersion did not much reduce in the aftermath of the crisis. Following this study, even though a concept difficult to measure, the Eurozone is still far from optimality as macroeconomic dispersion levels are higher than those in 1999. Many claim that a monetary union such as the Eurozone remains incomplete, and it cannot only last if accompanied by a meaningful 'central budget' or a so-called 'fiscal union' with the (at least partial) mutualisation of the MSs debts (see De Grauwe, 2012, 2013) and stabilisation mechanisms (Goodhart, 2011). Perhaps our proposal of parallel currencies could increase stability with no need of deep institutional reform.

The Eurozone crisis has revealed the abandonment of the Treaty of Maastricht (1992)'s consensus – essentially, no bailouts for Member States and no monetisation of the public deficit. This has proved politically unfeasible. Instead the European Commission and the Member States took several years to approve new mechanisms and tools to assist economies in crisis, as well as to adopt new institutions to monitor more closely macroeconomic and fiscal imbalances in the Eurozone (i.e. the so-called 'Fiscal Compact' and the 'Macroeconomic Imbalances Procedure'). These changes have signaled the choice of greater centralisation in addressing the current imbalances across Member States. This option has been criticised for the increased complexity and the regulatory burden added to the management of fiscal and macroeconomic disequilibria in the Eurozone, as well as for the lack of a credible economic discipline imposed on errant MS economies (see Wyploz, 2019 and Vaubel, 2020 for further details).

In a system of purely fiat money, stable money is the best central banks can achieve (Friedman, 1967). By aiming to keep the value of their money stable, central banks will at least refrain from using money to meddle in the national economy. The failure to enforce the Maastricht rules in the years before the crisis and to achieve even monetary stability in the years leading to the 2007 monetary crisis (see Congdon, 2017) - with a rate of money growth that nearly trebled the 'reference value' for M3 growth compatible with price stability as adopted by the ECB few years earlier (ECB, 1998) - , should incline one to examine another possibility: to let the amount of money be determined bottom up by introducing more competition in the money market, rather than top down by central bankers and governments. In this paper, we shall see how a regime of parallel currencies for failing countries of a monetary union approximates this ideal, and we will detail the conditions under which more monetary competition in the less flexible parts of the Eurozone may lead to better macroeconomic performance for the zone as a whole[1]; i.e. price stability and less output volatility along the business cycle. One of the major advantages of this option is the introduction of greater flexibility and discipline in the economy in a more decentralised Eurozone. This system will allow economies in crisis to address an adjustment in costs and prices more smoothly and in good time. In addition, once the crisis is over, competition

[1] Populist parties in Greece and Italy have recently proposed the issue of a new national currency (or IOUs) as a way to avoid fiscal discipline and allow for the monetisation of rampant government deficits. As we will show in sections 3 and 4, these proposals would lead to greater inflation in the national currency and thus to its expulsion from the market.

for the provision of stable money will determine ultimately the amount of money in circulation compatible with price stability. This monetary arrangement would carry no greater regulatory burden in the Eurozone in the form of new rules to monitor or enforce, nor the innovation of more instruments to assist economies in crisis; therefore avoiding the current political tensions among MSs and even their populations as regards the future of the Eurozone.

2. Previous discussions of parallel currency regimes

At the time of the discussion of the last stages of the European Monetary Union, Dowd and Greenway (1993) paved the way to modeling currency competition and in particular, discussed how realistic it would be to have a monetary system with two currencies in competition. Their model is based on the comparison of the utility functions resulting from the use of each currency, which (crucially) depend on the network effects and switching costs associated with the use of each currency. Contrary to what we propose, Dowd and Greenway (1993) with their model explain why currency competition or the introduction of a new (parallel) currency do not usually prevail in the economy and why it is optimal to have a single currency in the economy. Switching costs and inertia in the use of the (incumbent) currency are the major drivers of a single currency solution, and both factors have been at the core of the discussion in the literature on currency competition[2].

Lotz and Rocheteau (2002) also stress the high inertia of users of the old currency. They develop a search model of money to analyse the way successfully to launch a new fiat currency in a parallel currency scenario. Inertia would require specific policy actions from the government to encourage and even enforce the transition to the new currency. The inertia in the use of the incumbent currency, even when it shows a consistently inferior value than the alternatives, as well as the role of governments in supporting the transition to the stronger currency, can also be found in Luther

2 See the two sides of the argument in two seminal works by Hayek (1978 a, b) and Friedman (1984). For a more up to date discussion, Luther (2013) presents the case against Hayek's overconfidence in the ability of currency users to choose the more stable currency when the incumbent currency loses its value, due to large switching costs and network effects; and David (2013) for a rebuttal, stating that Hayek's position refers to the ability of businesses (and not ordinary people for ordinary transactions), via capital and savings transactions, to switch to the more stable money, a possibility that will discipline the government.

(2016) when he examines the prospects of Bitcoin as a successful alternative to government money.

We follow up on Dowd and Greenway's (1993) model by considering switching costs in ours; and we also share the assumption that monetary competition or indeed the introduction of a parallel currency system is frequently proposed when the incumbent currency has been very poorly managed for a long time. It is in times of monetary crisis and lack of trust in the national currency, indeed in the context of hyperinflation, when the launch of a new currency is considered as a policy option.[3] We examine the transition to the new currency as mainly driven by economic rather than political reasons. So, however important political considerations are in the economic integration literature, we do not set up our model in the context of the discussion on how to achieve monetary integration, but in that of a country with a national (legal tender) currency that has decided to add another one in competition with it.

In addition, unlike Dowd and Greenway (1993)'s, our model is not based on the running of parallel currencies under a fixed exchange rate. Instead, our model is based on the assumption that the two currencies float freely against each other; and proceeds to spell out the equilibrium conditions determining the market share of each currency. This free float is the distinguishing trait of our model, and thus of our policy prescriptions in section 6 below.

In section 3 we will briefly describe a parallel currency system. In section 4 we will set up a model of currency competition, with two currencies running in the economy under freely floating exchange rates. In section 5 we discuss the results of the equilibrium of the model and show under what conditions a system of parallel currencies can function stably and contribute to keeping inflation in check in both currencies. In section 6 we interpret the results of the model in two different policy scenarios, one corresponding to macroeconomic stability and another to extreme fluctuation in prices, either hyper-inflation or hyper-deflation. We conclude in section 7 by spelling out important policy implications for the easier integration national currencies expected to join the Eurozone.

[3] For example, Gomez and Helmsing (2008) discuss the stabilising benefits resulting from the adoption of new local currency systems in Argentina in the 1990s, at the time of the collapse of the value of the Argentinian peso. Parallel currency systems have also been suggested in the context of the launch of new supra-national currencies in Europe (Vaubel, 1978) and Asia (Eichengreen, 2006).

3. Parallel currencies under free floating exchange rates. The operation of the 'inverse Gresham's law'

The Great Recession and the subsequent Eurozone crisis have revealed two major flaws in the European Monetary Union (EMU), resulting from a lack of true commitment to the monetary orthodoxy demanded in the EMU treaties or from the absence of a commitment to establishing a fiscal union to back the currency (as proposed by De Grauwe, 2013). With the former not forthcoming and political consensus for the latter not existing, Europeans to look for a different kind of monetary arrangement for countries in crisis.

Our thesis is that launching the euro as a choice currency, instead of a single currency, would have made good many of its defects. At that time, the euro could have been introduced to run in parallel with the national currencies at fully flexible exchange rates.[4] This option was proposed at the time of the discussion of EMU by prominent academics (see Vaubel, 1978) and even as an official proposal by the British government in the early 1990s (see Phelan, 2015), but was disregarded as a policy option in the so-called 'Delors Report'. [5]

We hold it that this scheme of two free floating legal tender currencies running in parallel would have been better suited to a less-than-optimal European currency zone. Under the inverse Gresham law that 'good money displaces bad', the euro would have slowly replaced mismanaged national currencies.

As Mundell (1988) explained, there are two versions of Gresham's Law, direct and inverse, depending on whether fixed or flexible exchange rates are in place: the direct is expressed in the traditional statement that 'bad money displaces good' (when the rate of exchange is fixed or legal tender laws are enforced for the bad currency); the inverse, that 'good money displaces bad' (when the exchanges are free at the market rate). In the latter scenario, both currencies can co-exist stably when they float and capital moves freely, if the central banks in charge of each currency behave

[4] Our proposal of parallel currencies differs on one count from the 'hard ECU' proposed by the British government under John Major in the 1990s: namely, to introduce the euro as a transnational alternative to the national currencies allowing competition to provide monetary discipline. However, by fixing the rate of exchange of the 'hard ECU' once and for all with national currencies instead of letting it float as we propose, the European Monetary System would have been as unstable as the bimetallism in the second half of the 19th c.

[5] In contrast with *The Economist* (2015), we would make the issue of scrip money permanent but parallel with the euro. See also Jones, M. and O'Donnell, J. (2015); and Strupczewski, J. (2015).

conservatively. And they will tend to do so, for the very depreciation of the more inflationary currency will be an incentive to avoid over-issue: the inflationary currency would be less and less demanded in the market and thus its issuer would suffer a loss in seignorage revenues. This is what we will examine with our model.

Such an arrangement could be introduced in times of a crisis affecting Member States disproportionally (so-called asymmetric crises), or a crisis resulting from the unsustainable finances of a Member State. The risk of contagion to the rest of the area would be limited, thus avoiding a threat to the monetary architecture of the whole Eurozone. Also, the Euro-Member-State in crisis would find it easier to correct its course by having temporary recourse to its national currency. Further, the experience of Greece during the Eurozone crisis has shown that there is the danger that a euro Member State will run out of cash, since it cannot 'print' its own money and its only source of means of payment is a positive balance of payments. [6] With a parallel currency system, a failing Member States could temporarily have recourse to the local currency if they ran out of euros due to persistent balance of payments deficits [7]; while the fully flexible and free exchange rates between the common currency and the national currency would discipline Member States' Central Banks and Treasuries. Under our proposed parallel currency system, there would be no need for bailouts of Member States nor for fiscal integration (i.e., such as transfer payments to economies in crisis or the pan-European mutualisation of the debt) in order to preserve the euro and the overall EU integration project.

4. A model of parallel currencies

In this section we present a model in which two currencies, i and j, coexist in the economy. We will assume that the share of each currency is determined by the exchange rate of both currencies and switching costs. Our model focuses on the long-term equilibrium conditions and thus does not

[6] The growing external imbalances between the so-called core and peripheral countries were experienced even in the years prior to 2008: peripheral EMU economies such as Spain, Portugal and Greece ran a close to 10% current account deficit ratio to GDP on average in 2007, which amounted to a negative net foreign position of approximately 100% of GDP on average (see IMF (2012), pg. 2). In the same year Germany was running a higher than 5% current account surplus ratio to GDP and a positive net foreign position higher than the 20% of its GDP.

[7] Persistent external deficits are mainly due to the excessive public debt issue to balance the national Budget.

include interest rates as determinants of the currencies' respective market shares.[8] As our model shows, the long-term equilibrium is conditional on: (a) a low exchange rate elasticity of the demand for the competing currency; and (b) a low price level elasticity of the demand for either currency. The implicit assumption is that such stickiness reflects confidence in the proper behaviour of the respective central banks.

Definitions

Exchange Rate between the two currencies

Let $X_{i,j}$ denote the exchange rate between the two currencies; that is, how many units of currency j [the national currency] can be bought with one unit of currency i [the common currency]. [9]An increase in the value of $X_{i,j}$ means that one unit of currency i buys more units of currency j and thus represents an appreciation of currency i. Let the price level of currency i be denoted by P_i and the one for currency j by P_j. A high value of P_i in relation to P_j means that currency i has a smaller purchasing power than currency j and thus is less valued in the market (in other words, $X_{i,j}$ is small). The exchange rate is fully determined by price differentials in both currencies: an increasing price level for currency i, i.e. a decreasing purchasing power of currency i, will decrease the exchange rate as one unit of currency i is worth less units of currency j. Similarly, an increase in the price level for currency j will increase the exchange rate. If a the sensitivity of the exchange rate to the ratio of price levels (a is a positive real number), we have the following equation for the exchange rate between the two currencies in the market:

$$X_{i,j} = \left(\frac{P_j}{P_i}\right)^a \tag{1}$$

[8] This is why we do not need to distinguish between different time periods in the model.

[9] In this essay we analyse the demand elasticity of currencies as a whole without distinguishing the different elasticities for each of the functions of money. It has been observed that Zimbabweans continue to use the local currency for small day-to-day transactions, while preferring the US dollar for large transactions, as a numéraire, and as a store of value. Similarly, Argentineans, while continuing to use the peso for small purchases, prefer to use the US dollar as a numéraire for large transactions and as a store of value abroad.

Due to rigidities caused by determinants other than the price levels in both currencies, which we omit here for simplicity and our focus on the long term, the value of the exchange rate will be less than proportional to the ratio of the price levels in both currencies; thus, we expect $a \leq 1$.[10]

Let the exchange rate $X_{j,i}$ measure the number of units of i that can be bought with one unit of currency j. We can get $X_{i,j}$ from equation (1) by interchanging i and j. We observe that modelling the exchange rate as in (1) ensures the necessary condition that the two exchange rates $X_{i,j}$ and $X_{j,i}$ are inverses of each other, i.e. $X_{i,j} = X_{j,i}^{-1}$.

Switching costs

We denote the cost of switching from currency j to i by s_i and the cost of switching from currency i to j by s_j. We treat them as positive real numbers ($s_i, s_j > 0$), so a high value of s_i means high costs to switch from j to i. We assume that when it becomes more costly to switch from currency j to i than from i to j (i.e. if $\frac{s_i}{s_j} > 1$), then people will want to keep more currency i. This is because it is less costly given that they might at some point have to switch some money to the other currency. Therefore the higher the ratio of the switching costs $\frac{s_i}{s_j}$ the higher the share of currency i, ceteris paribus. Let us label the ratio of the switching costs as $s_{i,j} = \frac{s_i}{s_j}$. The reciprocal will be denoted as $s_{j,i}$.

Market share of a currency

θ_i and θ_j in (2) below denote the shares of currencies i and j, respectively; that is, θ_i of the economic transactions are made in currency i and θ_j are made in currency j. Consequently, in an economy with only two currencies available for market transactions, we have the following necessary requirements for θ_i and θ_j:

[10] However, $a > 1$ would also be conceivable, particularly in a hyper-inflation scenario when currency values are extremely sensitive to changes in inflation.

$$\theta_i, \theta_j \in [0,1]$$
$$\theta_i + \theta_j = 1 \tag{2}$$

The first condition above implies that the shares are real numbers between zero and one. The second one states that the aggregate share of the two currencies in the market must be one, as each transaction must be made in one of the two currencies available. In addition, any functional form for the shares of currencies i and j has to respect the two conditions in (2).

In our model the shares of the currencies will depend on the exchange rate and the switching costs between the two currencies. Specifically, the influence of the exchange rate $X_{i,j}$ on the share of currency i is positive as people would prefer a currency with a higher purchasing power; and people would want to keep more of the currency to which it is more costly to swap. We adopt equations (3) and (4) below to describe the dependence of the share of currency i and j on their respective switching costs and the exchange rate (see the appendix for a verification that this functional form respects the conditions in (2):

$$\theta_i = \frac{1}{1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}} \tag{3}$$

$$\theta_j = \frac{1}{1 + X_{j,i}^{-\nu} \cdot s_{j,i}^{-\mu}} \tag{4}$$

Where ν and μ are positive real numbers. These parameters measure how sensitive the currency share is to changes in $X_{i,j}$ and $s_{i,j}$, respectively. The higher their value the more sensitive the share of currency i reacts to changes in the exchange rate and switching costs. For instance, for a very high value of ν an exchange rate $X_{i,j}$ (just above one) might lead to near full displacement of currency j from the market. Whereas if ν is very small, the share of currency i does not react much to changes in the exchange rate. Again, since there are certain rigidities (inertia)[11] in the use of the currency that our model does not account for, we expect that ν and μ would not be very high and therefore the shares of the currencies would not be overly sensitive to changes in either the exchange rate or switching costs. We will discuss this in more detail in the last section of the paper.

Market share determinants

We now need to determine the factors affecting the exchange rates and thus the market share of a currency. Since our model focuses on the long-term relation between exchange rates and prices, we will also explain the major determinants of the price level according to the standard equation of the quantity theory of money:

$$M \cdot V = P \cdot Y \tag{5}$$

where M is the amount of money in circulation, V the income velocity of money, Y the income of the economy and P the price level.[12] In our model with two currencies we have similar equations for each currency:

$$\begin{aligned} M_i \cdot V_i &= P_i \cdot \theta_i Y \\ M_j \cdot V_j &= P_j \cdot \theta_j Y \end{aligned} \tag{6}$$

where V_i is the velocity of currency i and M_i the amount of currency i, similarly for j. For simplicity, we will assume that both velocities are equal $V_i = V_j$ and constant, in particular not influenced by the amount of money or other variables of the model. We will denote it by V. The difference between (5) and (6) is that instead of the total level of income in the economy, each of the two equations in (6) uses the share of the income exchanged in the respective currency; and

[11] For example, the market share will also depend on government regulations. The government may decide to pay its employees, pensioners, public contractors and social benefits in general in the State (preferred or incumbent) currency, which depending on the economy can represent from 35% to 50% of the total volume of transactions in the economy. However, we are not considering such factors in our model. These factors well justify a low value of $s_{i,}$.

[12] See the re-interpretation of the Quantity Theory of Money in Dowd (2014), pg. 56, where the importance of the market share of the two currencies is examined.

thus we have $\theta_i Y$ and $\theta_j Y$. We will treat the level of income Y as a constant as well, since our interest is in the shares of the parallel currencies and we do not intend to model economic growth.

Rearranging (6) we get to the following expressions for the price levels in both currencies:

$$P_{\mathrm{i}} = \frac{M_i \cdot V}{\theta_i Y}$$
$$P_{\mathrm{j}} = \frac{M_j \cdot V}{\theta_j Y} \tag{7}$$

According to equations (7) above the amount of money in circulation has an influence on the market share of each currency. Since both money velocity and the level of income are fixed[13], the effects of changes in the amount of each currency on their respective market share is explained by two different channels that move in opposite directions: on the one hand, a positive effect by which the increase in the amount of currency i, M_i, is followed by an increase in its market share θ_i so that (6) remains balanced. On the other hand, another option for (6) to remain balanced is that the increase in the amount of currency i leads to an increase in the price level in that currency (i.e. P_i increases). As a result of the increase in P_i, the exchange rate $X_{i,j}$ would decrease as currency i becomes less valuable, resulting in a decrease of the market share of currency i. We can see that the effect of an increase in M_i is ambiguous, so we need to assess which effect prevails: (a) If the influence of P_i on θ_i is so strong that an increase of P_i decreases the product $P_i \cdot \theta_i$, then the positive effect of M_i on θ_i will prevail. In fact, any increase in P_i would so strongly decrease θ_i that the right-hand side of equation (6) for currency i actually decreases instead of matching the higher level of M_i (compare to the discussion in section 5). (b) In the setting of a weak influence of P_i on θ_i, a higher level of M_i could be accounted for by an increase of P_i. The higher price level will be less than offset by its effect on θ_i, so that the right-hand side of equation (6) for currency i rises. We can suggest an economic interpretation for these two scenarios: (a) corresponds to a

[13] This is a very plausible assumption at least in the short term. Anyway, output is an exogenous variable in our model, and explained in the long term by factors we do not consider here. As per the velocity of money, even though it changes over time, over a sufficiently long time period, the ratio of cash holdings over income/wealth remains quite stable (see Congdon, 2005).

scenario where prices are rather sticky and the increase in the amount of the currency is not readily followed by an increase in prices in that currency; whereas scenario (b) occurs when prices are flexible enough to reflect changes in the amount of the currency.

A similar pattern explains the expected effects of changes in switching costs on the currencies' market share. We argued above that increased costs for switching from currency j to i should increase the share of currency i. We verify this in the appendix by assuming that the exchange rate remains constant. But this assumption is not realistic: if the share of currency i increases then the price level P_i and the exchange rate will also be affected (see equation (7) above). The higher exchange rate X_{ij} in turn will have a negative effect on the share of currency i. We will gauge these effects in the last section of the paper, which will depend on the sensitivity of θ_i to changes in P_i be similar the analysis above.

In order to see whether both currencies can coexist in our model and find out the overall effect of the input variables on the share of currency i, we will derive an expression of θ_i in terms of M_i, M_j and $s_{i,j}$. We start by inserting (7), the equation explaining the determinants of the price levels in both currencies, into our formula for the exchange rate (equation (1)):

$$X_{i,j} = \left(\frac{P_j}{P_i}\right)^a = \left(\frac{\frac{M_j \cdot V}{\theta_j Y}}{\frac{M_i \cdot V}{\theta_i Y}}\right)^a = \left(\frac{M_j}{M_i} \cdot \frac{\theta_i}{\theta_j}\right)^a = \left(\frac{M_j}{M_i} \cdot \frac{\theta_i}{1-\theta_i}\right)^a \tag{8}$$

Once we cancel Y and V and rearrange (8), we use the fact that the shares of the currencies must add up to one. Finally, we insert equation (8) into our equation (3) for the explanation of the share of currency i. The result is the following:

$$\theta_i = \frac{1}{1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}} = \frac{1}{1 + \left(\frac{M_j}{M_i} \cdot \frac{\theta_i}{1-\theta_i}\right)^{-a\nu} \cdot s_{i,j}^{-\mu}} \tag{9}$$

This equation implicitly links the share of currency i to the amount of money in both currencies and the switching costs. It seems plausible that the amount of money in both currencies affect the share of currency i (and thus the share of currency j). Since the amount of money can be seen as a variable at the discretion of the respective central banks (indeed an exogenous variable in our model determined by the monetary authorities), it will be interesting to examine how changes in the amount of money influence the share of currency i. In addition, we will also assess the net effect of a change in the switching costs. To do this, we will solve (9) for θ_i.

Market share equilibrium

Two trivial solutions for equation (9) can easily be identified, namely $\theta_i = 1$ and $\theta_i = 0$,which correspond to the situation where only currency i or only currency j runs in the market. It is not very surprising that these solutions exist; they simply correspond to the usual central bank monopolist situation of a single currency running in the economy under legal tender. The more interesting question is whether both currencies can coexist, in other words, whether there is a solution to equation (9) with θ_i not being one or zero. In order to address this question, we have to discard the two obvious solutions: our first step will be to isolate the $\theta_i = 1$ solution. We do this by multiplying the denominator of the right-hand side of (9) and rearranging as follows[14]:

$$\theta_i = \frac{1}{1 + \left(\frac{M_j}{M_i} \cdot \frac{\theta_i}{1-\theta_i}\right)^{-av} \cdot s_{i,j}^{-\mu}}$$

$$\Leftrightarrow \theta_i + \theta_i \cdot \left(\frac{M_i}{M_j}\right)^{av} \cdot \left(\frac{\theta_i}{1-\theta_i}\right)^{-av} \cdot s_{i,j}^{-\mu} = 1$$

$$\Leftrightarrow -(1-\theta_i) + (1-\theta_i)^{av} \cdot \left(\frac{M_i}{M_j}\right)^{av} \cdot \theta_i^{-av} \cdot s_{i,j}^{-\mu} = 0$$

$$\Leftrightarrow (1-\theta_i) \cdot \left(-1 + (1-\theta_i)^{av-1} \cdot \left(\frac{M_i}{M_j}\right)^{av} \cdot \theta_i^{-av} \cdot s_{i,j}^{-\mu}\right) = 0$$

[14] In the second step below, we have used the fact that we are not interested in the solution $\theta_i = 1$. In the third last step we make the assumption that $av \neq 1$. If in fact $av = 1$, then an easy computation shows that $\theta_i = 0$ and $\theta_i = 1$ are the only solutions to equation (10).

We have now isolated the solution $\theta_i = 1$ in the first factor. Since we are interested in a solution for θ_i different from one, we divide the last line of the above calculation by $(1-\theta_i)$. Then some more algebra steps will result in the following result:

$$(1-\theta_i)\cdot\left(-1+(1-\theta_i)^{av-1}\cdot\left(\frac{M_i}{M_j}\right)^{av}\cdot\theta_i^{-av}\cdot s_{i,j}^{-\mu}\right)=0$$

$$\overset{\theta_i\neq 1}{\Longleftrightarrow} -1+(1-\theta_i)^{av-1}\cdot\left(\frac{M_i}{M_j}\right)^{av}\cdot\theta_i^{-av}\cdot s_{i,j}^{-\mu}=0$$

$$\Leftrightarrow \left(\frac{1-\theta_i}{\theta_i}\right)^{av-1}\cdot\left(\frac{M_i}{M_j}\right)^{av}\cdot s_{i,j}^{-\mu}=0$$

$$\Leftrightarrow \left(\frac{1-\theta_i}{\theta_i}\right)^{av-1}=\left(\frac{M_i}{M_j}\right)^{-av}\cdot s_{i,j}^{\mu}$$

$$\overset{av\neq 1}{\Longleftrightarrow} \frac{1-\theta_i}{\theta_i}=\left(\frac{M_i}{M_j}\right)^{\frac{-av}{av-1}}\cdot s_{i,j}^{\frac{\mu}{av-1}}$$

$$\Leftrightarrow \theta_i\cdot\left(1+\left(\frac{M_i}{M_j}\right)^{\frac{-av}{av-1}}\cdot s_{i,j}^{\frac{\mu}{av-1}}\right)=1$$

$$\Leftrightarrow \theta_i=\frac{1}{1+\left(\frac{M_i}{M_j}\right)^{\frac{-av}{av-1}}\cdot s_{i,j}^{\frac{\mu}{av-1}}} \tag{10}$$

Equation (10) above is an explicit formula for the share of currency i as a function of the amount of money in both currencies and the switching costs. It is clear that (10) is a solution strictly between zero and one if the amounts of money M_i and M_j are non-zero, so it describes a situation where both currencies coexist.

We can also use (10) for θ_i to establish an explicit description of the price levels in both currencies in terms of the amount of currencies i and j and the switching costs. To do so, we insert (10), which determines the currency share θ_i, into equations (7) as follows:

$$\begin{aligned} P_i = \frac{M_i \cdot V}{\theta_i Y} &= \frac{M_i \cdot V}{Y} \cdot \left(1 + \left(\frac{M_i}{M_j}\right)^{\frac{-av}{aav-1}} \cdot s_{i,j}^{\frac{\mu}{av-1}}\right) \\ &= \frac{V}{Y} \cdot \left(M_i + M_i^{\frac{-1}{av-1}} \cdot M_j^{\frac{av}{av-1}} \cdot s_{i,j}^{\frac{\mu}{av-1}}\right) \end{aligned} \tag{11}$$

$$P_j = \frac{V}{Y} \cdot \left(M_j + M_j^{\frac{-1}{av-1}} \cdot M_i^{\frac{av}{av-1}} \cdot s_{i,j}^{\frac{-\mu}{av-1}}\right) \tag{12}$$

$$\frac{P_j}{P_i} = \frac{\frac{M_j \cdot V}{\theta_j Y}}{\frac{M_i \cdot V}{\theta_i Y}} = \frac{M_j}{M_i} \cdot \frac{\theta_i}{1-\theta_i} = \frac{M_j}{M_i} \cdot \left(\frac{M_i}{M_j}\right)^{\frac{av}{av-1}} \cdot s_{i,j}^{\frac{-\mu}{av-1}} = M_i^{\frac{1}{av-1}} \cdot M_j^{\frac{-1}{av-1}} \cdot s_{i,j}^{\frac{-\mu}{av-1}} \tag{13}$$

For the expression of P_j in (12) above we impose symmetry and exchange the currencies with respect to (11). For the ratio of price levels in (13), we insert (10) which determines the share of currency i as a function of the amount of money in both currencies and the switching costs.

Now that we have an explicit expression for θ_i in terms of M_i, M_j and $s_{i,j}$, we can assess the net effects of changes in these variables on θ_i and thus resolve the ambiguities mentioned in the section above.

We will only discuss the effects of M_i and $s_{i,j}$ on θ_i. By symmetry, the influence of M_j on θ_j will be of the same sign as the influence of M_i on θ_i; consequently, if we find a situation where an increase in M_i decreases θ_i, we will know that in this situation an increase in M_j will also decrease θ_j. Similarly, since θ_i and θ_j add up to one, a decrease in θ_j will mean an increase in θ_i; therefore if changes in M_i have a negative effect on θ_i, then it holds that changes in M_j have a positive effect on θ_i and vice versa. To see how M_i influences θ_i, we calculate the derivative of θ_i by M_i:

$$\frac{d\theta_i}{dM_i} = \frac{-s_{i,j}^{\frac{\mu}{av-1}} \cdot M_j^{\frac{-av}{av-1}} \cdot \frac{-av}{av-1} \cdot M_i^{\frac{-av}{av-1}-1}}{\left(1 + \left(\frac{M_i}{M_j}\right)^{\frac{-av}{av-1}} \cdot s_{i,j}^{\frac{\mu}{av-1}}\right)^2}$$
$$= \frac{av}{av-1} \cdot \frac{s_{i,j}^{\frac{\mu}{av-1}} \cdot M_j^{\frac{-av}{av-1}} \cdot M_i^{\frac{-2av+1}{av-1}}}{\left(1 + \left(\frac{M_i}{M_j}\right)^{\frac{-av}{av-1}} \cdot s_{i,j}^{\frac{\mu}{av-1}}\right)^2} \quad (14)$$

Given that the sign of the last fraction above is positive, we know that the sign of the derivative is determined by $\frac{av}{av-1}$: i.e. the derivative, and thus the effect of changes in the amount of currency i on its market share will be negative if $av < 1$ and positive if $av > 1$.

Since the price levels affect the share of the currencies, it is worth assessing the reaction of prices to the amount of money directly. Differentiating (11), (12) and (13) by M_i we arrive at the following expressions:

$$\frac{dP_i}{dM_i} = \frac{V}{Y} \cdot \left(1 - \frac{1}{av-1} \cdot M_i^{\frac{av}{av-1}} \cdot M_j^{\frac{av}{av-1}} \cdot s_{i,j}^{\frac{\mu}{av-1}}\right) \quad (15)$$

$$\frac{dP_j}{dM_i} = \frac{av}{av-1} \cdot \frac{V}{Y} \cdot \left(\frac{M_i}{M_j}\right)^{\frac{1}{av-1}} \cdot s_{i,j}^{\frac{-\mu}{av-1}} \quad (16)$$

$$\frac{d\left(\frac{P_j}{P_i}\right)}{dM_i} = \frac{1}{av-1} \cdot s_{i,j}^{\frac{-\mu}{av-1}} \cdot M_j^{\frac{-1}{av-1}} \cdot M_i^{\frac{2-av}{av-1}} \quad (17)$$

As with (14), the sign of $av - 1$ will be key to determine the sign of the relation between prices and money in (15): if $av < 1$, the derivative of the price level P_i by M_i will be positive, whereas the derivative of the price level P_j by M_i, and thus the derivative of the ratio of price levels by M_i, are negative. This result shows that increases in the amount of a currency will end up raising prices in that currency. However, if $av > 1$, then the sign of the derivative of P_i by M_i will depend specifically on the amount of money in each currency running in the economy and the switching costs. Regarding (16) and (17), the signs of the derivative of P_j by M_i and the derivative of the ratio of price levels by M_i are positive.

Before we start to interpret these results we will also calculate the derivative of θ_i with respect to $s_{i,j}$:

$$\frac{d\theta_i}{ds_{i,j}} = \frac{-\mu}{av-1} \cdot \frac{\left(\frac{M_i}{M_j}\right)^{\frac{-av}{av-1}} \cdot s_{i,j}^{\frac{\mu-av+1}{av-1}}}{\left(1+\left(\frac{M_i}{M_j}\right)^{\frac{-av}{av-1}} \cdot s_{i,j}^{\frac{\mu}{av-1}}\right)^2} \tag{18}$$

The last fraction is positive, so the sign is determined by $\frac{-\mu}{av-1}$: the derivative of θ_i by $s_{i,j}$ will be positive if $av < 1$ and negative if $av > 1$.

Finally, we calculate the derivative of the price levels with respect to the switching costs:

$$\frac{dP_i}{ds_{i,j}} = \frac{\mu}{av-1} \cdot \frac{V}{Y} \cdot M_i^{\frac{-1}{av-1}} \cdot M_j^{\frac{av}{av-1}} \cdot s_{i,j}^{\frac{\mu-av+1}{av-1}} \tag{19}$$

$$\frac{dP_j}{ds_{i,j}} = \frac{-\mu}{av-1} \cdot \frac{V}{Y} \cdot M_i^{\frac{av}{av-1}} \cdot M_j^{\frac{-1}{av-1}} \cdot s_{i,j}^{\frac{-\mu-av+1}{av-1}} \tag{20}$$

$$\frac{d\left(\frac{P_j}{P_i}\right)}{ds_{i,j}} = \frac{-\mu}{av-1} \cdot \left(\frac{M_i}{M_j}\right)^{\frac{1}{av-1}} \cdot s_{i,j}^{\frac{-\mu-av+1}{av-1}} \tag{21}$$

If $av < 1$, then the derivative of P_i by the switching costs is negative, the derivative of P_j by $s_{i,j}$ is positive as well is the derivative of the ratio of price levels by the switching costs. In the case of

$av > 1$ the derivatives have the opposite sign, so $\frac{dP_i}{ds_{i,j}} > 0$ whereas both $\frac{dP_j}{ds_{i,j}}$ and the derivative of the ratio of price levels by the switching costs are negative.

5. Interpretation of the results under different macroeconomic scenarios

In the section above we showed that whether av is greater or smaller than one is crucial in the interpretation of our results. a is the sensitivity of the exchange rate with respect to changes in the price levels; so that a high value of a means that a small change in the ratio of price levels has a high impact on the exchange rate. ν is the sensitivity of the currency shares with respect to the exchange rate; so that a high value of ν means that θ_i reacts very strongly to a change in the exchange rate. Therefore, the combined value of av is the sensitivity of the share of currency i with respect to the ratio of price levels: the higher it is the stronger θ_i decreases if $\frac{P_j}{P_i}$ decreases.

Currency share and money supply

Our model links prices, currency shares and the amount of money in each currency through the equations (6). Therefore, our model's predictions are based on the fact that after one variable changes and the balance of (6) is thus disturbed, the other variables must change in a way that restores equilibrium in (6).

In section 3 above we anticipated that a change in the amount of money M_i has a positive effect on θ_i directly through equation (6) and a negative effect indirectly through the increased price level. We will now asses which effect will prevail and in which circumstances.

Let us assume that we are in a scenario with low price sensitivity, i.e. $av < 1$, corresponding to a relatively stable economy. An increase in the amount of money M_i raises prices P_i according to (15), just as we would expect. It also decreases P_j and thus the ratio $\frac{P_j}{P_i}$ (via (16) and (17)). As a result, the market share of the more inflationary currency (θ_i) decreases as well, and consequently that of the more stable currency (θ_j) increases. But due to the relatively small price sensitivity in this fairly stable macroeconomic scenario θ_i does not decrease to a large extent. [15]

Next we consider the less likely case of high price sensitivity, i.e. $av > 1$. In such a highly price sensitive situation one would assume some extreme behaviour in the economy should the determinants of prices and exchange rates vary. Indeed, such extreme effects can be explained along the lines of our model. However, we will see that such an explanation would prevent us from ever again reaching equilibrium in equations (6), as one of the two currencies will completely displace the other from the market. Therefore, we can interpret the, at first sight surprising, results of our model in this highly volatile scenario as the policy description of what the monetary authorities would need to do in order to maintain a stable two currency economy, even when price sensitivity is high, so that we can avoid hyperinflation in either currency. We will discuss these monetary policy prescriptions in section 5.

Let us first describe intuitively what would happen if $av > 1$: if the amount of money in currency i increases, then one would expect the price level P_i to increase too, similar to what happened in the case of low price sensitivity described above. But since θ_i is now very price sensitive, the result of such an increase in P_i would be a sharp drop in the share of currency i. Overall the right-hand side of equation (6) for currency i would decrease. Since there is more currency i in the market than that needed for the share of the output that is traded in currency i, prices in currency P_i would rise even further. Now we see that we are in a sort of a vicious circle leading to greater and greater inflation, and we cannot reach equilibrium anymore and as a result θ_i would fall continuously. In fact, we are under a hyperinflation spiral: eventually, currency i would nearly disappear while prices in currency i become arbitrarily large and thus we have hyperinflation in currency i. This means that θ_j gets close to one (and thus becomes the virtual monopolist currency in the market) and that P_j would fall, which again makes θ_i decrease further more. These outcomes would only

[15] Following (7), the quantity equation for currency i balances with an increased amount of currency i, which is offset by a substantial increase in P_i and a relatively small decrease in θ_i. The quantity equation for j remains balanced on the same level as before; in this case, the decrease in P_j is counteracted by the increase of θ_j.

be reverted should the monetary authority issuing currency i cut down the amount of its currency in circulation. [16]

Currency share and switching costs

Under a scenario with low price sensitivity of the currency share ($av < 1$), we turn now our focus to the effect of changes in the switching costs on each currency's market share. It is important to note that switching costs affect the currency share directly. In contrast to what happened as a result of changes in the amount of money, now changes in the price levels do not play a role in the explanation of the change in the shares of the currencies, but are rather a consequence of it.

When the switching costs change in a way that $s_{i,j}$ rises, the share of currency i will be higher for all ratios of price levels, as equation (22) in the appendix shows. So the rise in $s_{i,j}$ increases the right hand side of equation (6) for currency i. Since now more goods are traded in currency i, while the amount of currency i stays the same, the price level P_i decreases. In this case, the increased use of currency i, the supply of currency i being steady, will increase its purchasing power. Applying a similar argument to the equation for j we get that the price level P_j rises as there is an excess of currency j and thus it loses purchasing power. Consequently, the ratio of price levels increases (see (21)), which has an additional positive effect on the market share of currency i. However, due to the low price sensitivity assumed in this scenario, this positive effect is small and the resulting increase of the ratio of price levels has an even smaller positive effect on θ_i. The whole process thus converges to equilibrium.

Under a scenario with high price sensitivity, i.e. $av > 1$, an increase in $s_{i,j}$ will not lead to an equilibrium. By (22) at each ratio of price levels, the share of currency θ_i will be higher. As

[16] The described behaviour, while being economically plausible, prevents the equations in (6) from ever getting balanced again (in fact, θ_i tends to 0 asymptotically, but does not reach 0). In the real world, where arbitrarily small shares of currency i are not possible, currency i will of course eventually disappear entirely. We would be back in the much more standard situation of a single currency in the market. While we can argue along the lines of our model to explain the intuitive domination of currency i by currency j in the high price sensitivity scenario, our model does not predict it formally because we explicitly excluded the single currency case in the derivation of (10). We discuss this in section 5.

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described above, we now have not enough currency i to pay for the goods traded in that currency and P_i decreases (similarly P_j increases). Since θ_i is now very price sensitive, this change in prices increases θ_i quite strongly, which overcompensates the decrease in P_i and thus the equilibrium in the market with two currencies cannot be achieved (equation (6) cannot be re-balanced), as the right hand side is always too large. Here we have a hyper-deflation in currency i, whose market share will continue to rise and thus displaces currency j further and further along time, eventually leading to a currency monopoly (currency i will disappear from circulation). However, as mentioned in the previous section, this is the intuitive reasoning along our model in a high price sensitivity scenario but not its formal prediction (see note 17).

6. Monetary policy under extreme macroeconomic conditions

According to our analysis above, under a high price sensitivity scenario, changes in the amount of money or in the switching costs may well lead to the collapse of the two-currency system, ending in a currency monopoly with either hyper-inflation or hyper-deflation in one of the currencies. The issuers of the currency would have to be very careful in order to maintain a stable two-currency system in this very volatile economy.

Our model, however, makes counter-intuitive predictions in the high price sensitivity scenario. As discussed above, this is to be expected, since the model is built on the assumption that both equations in (6) hold and both currencies coexist. As the intuitive arguments in section 4 show, this is not plausible in a high price sensitivity scenario (either we let one currency completely displace the other, i.e. $\theta_i = 0$ and $\theta_j = 1$, which would allow the equations in (6) to be balanced, or we let θ_i only tend to zero asymptotically, but then there will be no equilibrium). This might suggest that our model is not suitable for this scenario.

But the predictions of our model are not without value in this high price sensitivity scenario. We can interpret them as a policy prescription of what would need to happen for both currencies to keep their value and coexist even in a high price sensitivity setting. In that sense they can be interpreted as guidance to monetary authorities willing to stabilise a two-currency economy when prices are very sensitive to small changes in the economy.

Let us now turn to these counter-intuitive predictions of the model. By (16) and (17) we see that the ratio of prices $\frac{P_j}{P_i}$ increases, as does P_j, when the amount of currency i increases. In addition, the share of currency i thus increases (14). This - at first sight counter-intuitive - prediction is consistent with equation (6) above but only under a high price sensitivity scenario: If, for the sake of the argument, $\frac{P_j}{P_i}$ decreased there could be no equilibrium as we concluded in the previous section. Only when $\frac{P_j}{P_i}$ increases the share of currency i will sharply increase.[17] Crucially, the balancing of the two equations hence does not come from the immediate effect which an increase of M_i has on the price levels, but from the indirect effect that changes in the price levels have on the market shares of the currencies. This indirect effect will be more prominent the more sticky prices are in the short run, resulting in an increase in the market share of the (inflated) currency. The same rationale also helps us to understand the result for the switching costs: If $s_{i,j}$ increases, so does θ_i for every ratio of price levels; but the ratio of the price levels changes as well. In fact, as equations (19) to (21) tell us, the change in the price levels is of the sign that apparently worsens the imbalance in (6), for instance P_i rises. The key point here is that this decrease in the ratio of price levels has a strong negative effect on θ_i which outweighs the positive effect of $s_{i,j}$, so that overall the share of currency i decreases. Thus, (6) remains balanced with higher prices P_i and lower share θ_i , and vice versa for currency j.

One might argue that the indirect effects that guarantee a stable two-currency economy in the case of high price sensitivity are hard to achieve in the real world. The intuitive scenario of hyper-inflation or hyper-deflation in one currency seems more likely. However, our model gives an indication of potential mechanisms available to avoid this collapse to one currency and might therefore help central banks to carefully maintain parallel currencies, even under a very volatile scenario as that with high price sensitivity.

[17] That way the right-hand side of (6) for currency i can increase and thus accommodate the increase of the amount of money of currency i. For currency j the increase in the prices P_j and the decrease of θ_j keep equation (6) balanced as well.

7. Policy conclusions

What the analysis of our two-currency model suggests is that there is an equilibrium in the market for the coexistence of both currencies. However, this equilibrium can only be maintained under what we have called a stable macroeconomic scenario; one in which the sensitivity of the market share of the currencies to changes in prices in both currencies is not high (as we presume changes in inflation in both currencies will be rather small). In this scenario, changes in the market share of one currency will be ultimately determined by the relative changes in the amount of that currency and thus in relative changes in prices in one currency relative to the other. Therefore, the less inflationary currency will gain market share over the other currency.

This would allow the members of a monetary union to keep a sound national currency along with the common currency; a monetary system that offers the incentives to keep the purchasing power of both currencies. Such a monetary system would offer more policy options for Member States suffering a euro liquidity crisis, as they could use their own currency for temporary relief. The flexible exchange rate would give timely warnings when this temporary remedy was being abused.

The model is also useful to explain under which conditions the two-currency economy would collapse into a single currency as the failed currency would suffer from hyper-inflation. Crucially the full expulsion of one currency by the other will happen in a highly unstable macroeconomic scenario, where agents' demand of each currency is very sensitive to changes in relative prices in both currencies. In this high price sensitive scenario, an increase in the switching costs to favour the use of one of the currencies (i.e. the government's preferred currency) would only lead to inflation in that favoured currency and very quickly to its expulsion from the market.

The results from our model show the conditions under which a parallel currency system will discipline the issuers of the currencies and thus to maintain the purchasing power of the currencies. It also discourages governments (or private issuers) from inflating one of the currencies as a means to raise seignorage, as this policy will soon end up in the displacement of the currency from the market; and any efforts by the issuer of the inflated currency to force agents to use their currency (i.e. by increasing the costs of switching to the other, more stable, currency) will also accelerate inflation in that currency and thus its expulsion from the market.

The European Union aspires to having all Member States use the euro as their currency: eight are still missing. Sweden in a referendum rejected joining; Denmark has opted out: but neither would have difficulty to join; our parallel currency scheme would ease the transition. Bulgaria, Croatia, and Romania, though willling, will have great difficulty in fulfilling the conditions of the Stability and Growth Pact. The Czech Republic, Hungary, and Poland show a different attitude, for the opposition to adopt the euro as their single currency is quite strong in these countries. For all these six Member States our scheme of the local currency running in parallel with the euro with freely floating exchanges would certainly facilitate their joining: short run difficulties could be addressed by temporarily easing monetary policy in the local currency, but the pressure would be there to stop prolonged depreciation.

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Appendix

Let us verify that θ_i reacts to the exchange rate and the switching costs as we describe above. We do this by taking partial derivatives with respect to $s_{i,j}$ and $X_{i,j}$. [18]

$$\frac{\partial \theta_i}{\partial s_{i,j}} = \frac{\mu \cdot X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu-1}}{\left(1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}\right)^2} > 0 \qquad (22)$$

So if the ratio of switching costs increases and the exchange rate remains the same, then the share of currency i increases as desired. Similarly,

$$\frac{\partial \theta_i}{\partial X_{i,j}} = \frac{\nu \cdot X_{i,j}^{-\nu-1} \cdot s_{i,j}^{-\mu}}{\left(1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}\right)^2} > 0$$

So the share of currency i increases with $X_{i,j}$ provided that the switching costs remain constant.

The first condition specified in (2) is satisfied since the expression of θ_i is clearly between zero and one. It remains to check that the two shares add up to one. This is a consequence of the fact that for every real number $x \neq -1$ we have

$$\frac{1}{1+x} + \frac{1}{1+x^{-1}} = \frac{1}{1+x} + \frac{x}{x+1} = \frac{1+x}{1+x} = 1$$

This computation with $x = X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}$ reads:

$$\begin{aligned}\theta_i + \theta_j &= \frac{1}{1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}} + \frac{1}{1 + X_{j,i}^{-\nu} \cdot s_{j,i}^{-\mu}} = \frac{1}{1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}} + \frac{X_{j,i}^{\nu} \cdot s_{j,i}^{\mu}}{X_{j,i}^{\nu} \cdot s_{j,i}^{\mu} + 1} \\ &= \frac{1}{1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}} + \frac{X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}}{X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu} + 1} = \frac{1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}}{1 + X_{i,j}^{-\nu} \cdot s_{i,j}^{-\mu}} = 1\end{aligned}$$

[18] Note that when taking a partial derivative, the other variables are treated as constants. In section 3.2 we see that there are interdependencies that might change the situation when taking the total derivatives, which considers all interdependencies.

We used that $X_{i,j} = X_{j,i}^{-1}$ and $s_{i,j} = s_{j,i}^{-1}$ when going from the first to the second line. Also note that since $X_{i,j}$ and $s_{i,j}$ as well as their reciprocal are positive, so that the above restriction $x \neq -1$ is satisfied. It is worth noting that if the exchange rate is one and the costs for switching from i to j and from j to i are the same, that is $s_{i,j} = 1$, then the model plausibly predicts that both currencies have the same share of 0.5.