SAF/No.29/March 2020

Studies in Applied Finance

ON THE QUEST FOR THE HOLY GRAIL OF DIVERSIFICATION

Hesam N. Motlagh and Steve H. Hanke

Johns Hopkins Institute for Applied Economics,
Global Health, and the Study of Business Enterprise

On the Quest for the Holy Grail of Diversification

By Hesam N. Motlagh and Steve H. Hanke

About the Series

The *Studies in Applied Finance* series is under the general direction of Professor Steve H. Hanke (hanke@jhu.edu), Founder and Co-Director of The Johns Hopkins Institute of Applied Economics, Global Health, and the Study of Business Enterprise, and Dr. Chris[illegible] Culp (christopher.culp@jhu.edu) and Dr. Hesam Motlagh (hnekoor1@jhu.edu), Fellows at the Institute for Applied Economics, Global Health, and the Study of Business Enterprise.

About the Authors

Hesam Motlagh is a Fellow at the Johns Hopkins Institute for Applied Economics, Global Health, and the Study of Business Enterprise, a Visiting Scholar in the Johns Hopkins Department of History, and an MBA candidate at the Stanford Graduate School of Business. As a Pear Venture Capital Fellow in Silicon Valley, he is an entrepreneur focusing on applying artificial intelligence to pharmaceutical and biotechnological drug development. At the Institute, he was a lecturer in Professor Steve Hanke's Applied Economics and Finance course for two years culminating in a book entitled, "Security Analysis: A Probabilistic Approach." Previously, he was a quantitative research analyst at Croft-Leominster Investment Management in Baltimore, where he focused on equity valuation and portfolio theory. He has authored or co-authored numerous research articles and book chapters, including a review that was highlighted on the cover of *Nature* magazine. His current research interests at the Institute include portfolio optimization, machine learning algorithms as they apply to asset pricing, and value investing. He holds a Ph.D. in Molecular Biophysics from Johns Hopkins University, and two B.S. degrees: one in Biochemistry and a second Mathematics & Statistics both from Miami University, Ohio.

Steve H. Hanke is a Professor of Applied Economics and Founder & Co-Director of the Institute for Applied Economics, Global Health, and the Study of Business Enterprise at The Johns Hopkins University in Baltimore. He is a Senior Fellow and Director of the Troubled Currencies Project at the Cato Institute in Washington, D.C., a Senior Advisor at the Renmin University of China's International Monetary Research Institute in Beijing, a Special Counselor to the Center for Financial Stability in New York, a contributing editor at *Central Banking* in London, and a regular contributor to the Wall Street Journal's Opinion pages. Prof. Hanke is also a member of the Charter Council of the Society of Economic Measurement and of Euromoney Country Risk's Experts Panel.

In the past, Prof. Hanke taught economics at the Colorado School of Mines and at the University of California, Berkeley. He served as a Member of the Governor's Council of Economic Advisers in Maryland in 1976-77, as a Senior Economist on President Reagan's Council of Economic Advisers in 1981-82, and as a Senior Advisor to the Joint Economic Committee of the U.S. Congress in 1984-88. Prof. Hanke served as a State Counselor to both the Republic of Lithuania in 1994-96 and the Republic of Montenegro in 1999-2003. He was also an Advisor to the Presidents of Bulgaria in 1997-2002, Venezuela in 1995-96, and Indonesia in 1998. He played an important role in establishing new currency regimes in Argentina, Estonia, Bulgaria, Bosnia-Herzegovina, Ecuador, Lithuania, and Montenegro. Prof. Hanke has also held senior appointments in the governments of many other countries, including Albania, Kazakhstan, the United Arab Emirates, and Yugoslavia.

Prof. Hanke has been awarded honorary doctorate degrees by the Bulgarian Academy of Sciences, the Universität Liechtenstein, the Universidad San Francisco de Quito, the Free University of Tbilisi, Istanbul Kültür University, Varna Free University, and the D.A. Tsenov Academy of Economics in recognition of his scholarship on exchange-rate regimes. He is a Distinguished Associate of the International Atlantic Economic Society, a Distinguished Professor at the Universitas Pelita Harapan in Jakarta, Indonesia, a Professor Asociado (the highest honor awarded to international experts of acknowledged competence) at the Universidad del Azuay in Cuenca, Ecuador, a Profesor Visitante at the Universidad Peruana de Ciencias Aplicadas (the UPC's highest academic honor), and the ECAEF Gottfried von Haberler Professor at the European Center of Austrian Economics Foundation in Liechtenstein. In 1998, he was named one of the twenty-five most influential people in the world by *World Trade Magazine*.

Prof. Hanke is a well-known currency and commodity trader. Currently, he serves as Chairman of the Supervisory Board of Advanced Metallurgical Group N.V. in Amsterdam and Chairman Emeritus of the Friedberg Mercantile Group, Inc. in Toronto. During the 1990s, he served as President of Toronto Trust Argentina in Buenos Aires, the world's best-performing emerging market mutual fund in 1995.

Prof. Hanke's most recent books are *Zimbabwe: Hyperinflation to Growth* (2008) and *A Blueprint for a Safe, Sound Georgian Lari* (2010), *Juntas Monetarias para Paises en Desarollo* (2015), *Currency Boards for Developing Countries: A Handbook* (2015), and *Gelişmekte Olan Ülkeler İçin Para Kurullari El Kitabi* (2019).

Prof. Hanke and his wife, Liliane, reside in Baltimore and Paris.

Abstract

For over a century, academics and market participants have studied two fundamental aspects of finance: asset valuation and portfolio construction. Despite the rich body of literature, studies tend to focus on only one of these concepts and use relative return volatility over a short time period (i.e. days, weeks, or months) as the investment risk metric. In contrast, certain investors utilize fundamental analysis and rely on mean-reversion, which occurs over a much longer timescale (three to five years). Here we argue that the proper risk metric for longer time horizon investments is the variance of the cash flows the asset generates instead of short timescale market price volatility. Accordingly, the values obtained for return versus risk take on a fundamentally different form. When integrated with Modern Portfolio Theory, this leads to a different efficient frontier that does not require as much diversification to obtain optimal portfolios – i.e. only six to eight assets. Although there is substantial empirical evidence for small portfolios in value investors, private-equity firms, and hedge funds, this represents (to the authors' knowledge) the first quantitative formalism to reach such a conclusion. The authors demonstrate this concept in practice with a discounted cash flow model in conjunction with a Monte-Carlo simulation to determine the probable variance in cash flows. These results suggest that the integration of portfolio construction with fundamental analysis may minimize the risk of large losses, while still creating the opportunity for profits without dampening the effect through over-diversification. These results call into question the over-diversification of fund portfolios and suggest a general strategy for long-term value investing.

Acknowledgements

The authors thank Ed Li for comments on an earlier draft of this paper, and Spencer Ryan for his edits.

Introduction

Like the quest for the Holy Grail, the search for the holy grail of portfolio diversification has proceeded with great enthusiasm and many crusaders. As it turns out, the search has been underway for centuries.

Indeed, even William Shakespeare hit on the topic of diversification in *The Merchant of Venice* when Antonio states (Shakespeare (1600)):

> "… I thank my fortune for it: My ventures are not in one vessel trusted, nor in one place, nor does my wealth depend upon the fortune of this present year. Therefore, my ventures do not make me somber." *Act 1, Scene 1*

Not only does Shakespeare articulate the intuition behind diversification, but implicitly suggests the idea of covariance – i.e. diversifying investments requires assessment of how prices will fluctuate individually and with respect to one another in the future.

When it comes to the diversification of securities, Henry Lowenfeld, a Polish émigré who settled in London, was one of the first to systematically deal with the problem. It was in London that he established the Universal Stock Exchange. He was also an early enthusiast of portfolio diversification on a grand scale. In his 1909 book, carrying the imposing title: Investment: An Exact Science, he proposed the theory of "Geographical Distribution of Capital", emphasizing not only a division of capital among stocks and bonds to reduce company-specific risks, but a division of securities among countries and regions to reduce market-specific risks (Goetzmann (2016)). This theme remains as vibrant today as it was when introduced by Lowenfeld (Sindreu (2020)).

Following Lowenfeld, the noted economist Irving Fisher put his stamp of approval on diversification (Fisher (1922, 1930)). In his 1922 book, The Making of Index Numbers, Fisher found that the variation in a price index declined rapidly as the number of individual prices in an index approached 20. From this insight, Fisher concluded that by picking 20 stocks, most of the risk of holding an individual stock would be eliminated.

But, it wasn't until Nobelist Harry Markowitz published "Portfolio Selection" in 1952 that the diversification grail was "found." Indeed, with Markowitz, diversification was carved in stone. Markowitz was then taken a step further by another Nobelist, William Sharp (Sharp (1963)).

Markowitz (Markowitz (1952)) and Roy (Roy (1952)) were the first to provide formalisms for *how* to diversify investments by utilizing mathematical rigor. Simply put, the portfolio construction process involves two major steps. First is the development of future performance expectations. Second is the choice of portfolio. The birth of modern portfolio theory (MPT) is credited to Markowitz for ascribing a mathematical formalism manifesting that the investor desires return while minimizing risk (Markowitz (1952); Markowitz (1959); Markowitz

(1999)). This has led to the so called "E-V" rule. In his seminal work, Markowitz tentatively suggests that return and risk could be defined as historical relative percent return and variance for stocks. In parallel, Sharpe addressed similar questions and proposed the CAPM model (Sharpe (1963); Sharpe (1964)). At this juncture, the asset pricing literature parted from the portfolio theory literature and there was a foregone conclusion in both fields: relative return variance is the proper risk metric as Markowitz had initially suggested (Campbell (1991); Elton and Gruber (1977); Evans and Archer (1968); Fama and French (1993); Francis and Kim (2013); Lakonishok, Schleifer and Vishny (1994); Sharpe (1964); Statman (2004)).

As portfolio diversification developed, more direct questions were asked. Specifically, how many stocks does a diversified portfolio contain? Evans and Archer's early study determined that approximately eight stocks were enough to mitigate unsystematic risk (Evans and Archer (1968)). Further work from Statmen and Elton further developed this work and concluded that 30 and even up to 300 securities are required to diversify risk (Elton and Gruber (1977); Statman (1987); Statman (2004)). The conclusion from the culmination of academic research is clear - it takes a significant number of securities to diversify. This conclusion has been so foundational that it has even been incorporated into the modern regulatory structure of mutual funds (Brealey, Myers and Marcus (2015); Francis and Kim (2013)).

But, there can be costs to diversification. Indeed, Robert Aliber has cautioned that excessive costs can accompany diversification (Aliber (2011)). However, these costs can be mitigated by passive investing in index funds that offer the investor a product that allows them to have their cake and eat it, too. With the recent flow of capital from active to passive management (Wigglesworth (2020)) it seems to almost be a self-fulfilling prophecy: if one needs to minimize risk, then the objective is to diversify to the point of indexation.

While many, if not most, think the holy grail of diversification has been found, there are serious doubters. For example, Warren Buffett, who has asserted that "[d]iversification is a protection against ignorance. [It] makes very little sense for those who know what they're doing" (Lowe (1997)). Buffett likes to explain the costs of over-diversification with remarks like: "If you have a harem of 40 women, you never get to know any of them very well" (Lowe (1997)). Buffett's long-time partner Charlie Munger also believes that diversification is less of a Holy Grail than a crackpot idea. Munger goes so far as to say that "the idea of excessive diversification is madness . . . We believe that almost all good investments will involve relatively low diversification" (Kaufman (2008)). As for diversification as it relates to indexation, Munger has this to say, "I have more than skepticism regarding the orthodox view that huge diversification is a must for those wise enough so that indexation is not the logical mode for equity investment. I think the orthodox view is grossly mistaken" (Kaufman (2008)).

Empirical work on the performance of mutual funds lends support to Buffett and Munger's "small is beautiful" idea. This research shows that concentrated mutual funds tend to outperform diversified funds and have persistent returns beating their respective benchmarks (Carhart (1997); Cremers and Petajisto (2009); Petajisto (2013); Yeung, Pellizzarti, Bird and Abidin (2012)). This should come as no surprise for active portfolio managers; to leverage their stock picking ability, concentration is required to obtain excess return above their respective benchmark. However, we still arrive at a portfolio construction method that is at odds with the bulk of portfolio theory. What could possibly explain this paradoxical observation? Do concentrated investors take on excessive risk to construct inefficient portfolios or are there other possible explanations?

This study addresses these questions by returning to one of the fundamental assumptions on the subject matter: risk. What is the proper risk metric for an investor? Here we show through theory and application that changing the risk metric not only allows for a different fundamental form of the efficient frontier, but readily leads to concentrated portfolios. Given that cash flows fundamentally determine asset prices (Brealey, Myers and Marcus (2015); Fisher (1974)), we argue that *cash flow variance* and not the *variance of relative returns* should be used as the measure of risk. We explore these results in three main parts. The first section analyzes the implications of changing risk on a theoretical level and finds that the efficient frontier takes a fundamentally different form when we consider cash flows and the exact correlations between companies as opposed to approximations from previous studies. As a result, we yield portfolios that are not only smaller in terms of the number of securities but can have cash flow in monetary value be the risk measure. We find that this makes more intuitive sense to an individual investor – i.e. one who is trying to construct a portfolio that optimizes cash flow generation given a specified risk in the form of absolute loss. The second section presents an example of how to perform such an analysis in practice and reaches an identical conclusion. We employ discounted cash flow models coupled with Monte-Carlo simulations that yield the cash flow variance of stocks to develop a portfolio for the S&P 500 similar to previous work in the portfolio theory literature. The final section ties together the conclusions we have drawn, discusses implications of this study, and areas of future research. These results suggest that the integration of portfolio construction with asset pricing may minimize the risk of large losses, while still creating the opportunity for profits without the dampening effect of over-diversification. These results call into question the over-diversification of mutual funds and suggest a general strategy for portfolio construction.

I. Risk and Return Shape Changes the Efficient Frontier Diversity

In this section we consider two thought experiments that demonstrate the determinants of small portfolios that also lie on the efficient frontier. First, we use monthly returns to determine risk and return of efficient portfolio sizes similar to previous studies and obtain identical results. However, we find that using actual stock correlations (versus assumed constant correlations between each pair of stocks and constant percentage composition (Evans and Archer (1968); Statman (1987); Statman (2004))) decreases portfolio sizes in contrast to what has been previously reported. This leads us to conclude that the simplifying assumption of correlation and percentage composition may have artefactually increased optimal portfolio sizes in previous studies. Second, we address how the shape of the efficient frontier needs to change to achieve small portfolios at a purely theoretical level. We find that if the risk metric makes it possible to find slightly higher returns for lower risk, the portfolio size decreases precipitously.

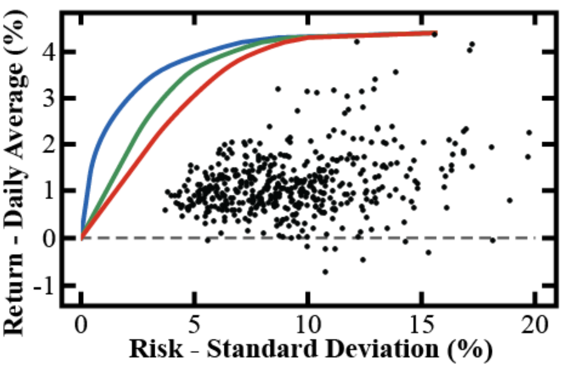
We first considered the monthly return of every stock in the S&P 500 and the statistics from the past decade (only stocks with 5 years of data are considered for robust statistics; see the end of the article to access to the code used to generate all the results presented). We use average relative return versus relative return standard deviation as return versus risk respectively. These results are shown in Exhibit 1, Left. Overlaid on this plot are three different efficient frontiers determined by mean-variance optimization using Modern Portfolio Theory as elaborated on below (Elton (2010); Francis and Kim (2013)).

Previous studies make the simplifying assumption that the correlation is constant and identical between all pairs of stocks to make the calculations more tractable (Francis and Kim (2013); Statman (1987); Statman (2004)). These studies yield estimates that range from 30 up to 300 stocks for an efficient portfolio. To ensure our methodology can reproduce these results and to test the significance of this assumption, we constructed efficient frontiers under three different conditions: no correlation between equities, a constant correlation of 0.08 as previously used (Elton (2010); Statman (2004)), and actual correlations between stocks shown as blue, green, and red lines in Exhibit 1, Left respectively.

It is clear from the data that the assumption of correlation changes the shape of the efficient frontier. Specifically, zero and constant correlation result in reduced risk for expected return as is demonstrated by the blue and green curves being shifted left in Exhibit 1. When we consider the actual correlations between these equities, we obtain efficient portfolios that are riskier than those obtained from the simplifying assumptions denoted by the red line in Exhibit 1 – i.e. for a given return, we have more risk. To address whether this assumption changes the portfolio size, we analyzed the distribution of portfolio sizes across the entire efficient frontier (Exhibit 1, Right). Indeed, when we use the simplifying assumption of zero or constant correlation, we obtain results that are consistent with previous studies. Specifically, the average portfolio size on the efficient frontier is approximately 190 and 43 equities for zero

Exhibit 1

Portfolio construction using daily return variance as risk yields smaller portfolios when correlations of stocks are utilized



Method	Mean	Std. Err.	Min	Q1	Median	Q3	Max
Real	17.4	6.0	1.0	13.5	20.5	22.0	22.0
Constant	40.3	22.1	1.0	18.8	43.0	62.0	63.0
Zero	190.0	190.0	1.0	34.5	172.0	454.0	467.0

and constant correlation respectively (Elton and Gruber (1977); Elton (2010); Evans and Archer (1968); Statman (1987); Statman (2004)).

When we use the real correlations, we obtain a precipitous decrease in the average portfolio size to approximately 17 stocks (Exhibit 1, Right Table). This result leads us to conclude that the simplifying assumptions used in previous studies lead to an artefactual increase in the number of securities required to diversify unsystematic risk. However, the value of ~17 assets in the portfolio is still large when compared to certain portfolios empirically utilized by certain value investors. Is this due to increased risk taken on by these portfolios, or are there other contributions that may yield smaller portfolios?

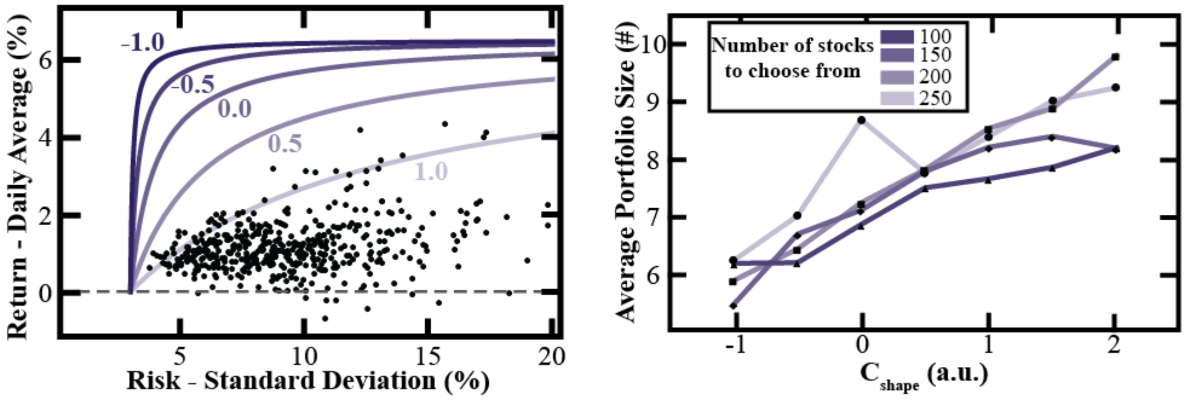
In our second thought experiment, we consider the general shape of the risk versus return curve for stocks which may change when we consider a different risk metric. We will simply ask the question: how does the shape of the efficient frontier need to change to reduce the overall number of assets in efficient portfolios? We will ascribe a simple empirical model to restrain the space sampled by potential assets to own for these purposes:

$$Return(Risk) = \frac{R_{Max}(Risk - Risk_{Min})}{10^{C_{shape}} + Risk - Risk_{Min}}$$

where R_{max} is the maximum return obtained by an asset (i.e. at infinite risk), $Risk_{Min}$ is the minimum value of risk sampled by an individual asset at approximately zero return, and C_{shape} is a parameter that describes the shape of the bound of risk versus return. We do not claim that this simplified model of restraints can be mechanistically interpreted, however, it does have a few advantages when exploring changes in the shape of the sample space for our purposes. First and foremost, it describes the overall shape of risk versus return sampled by real data (Exhibit 2, second lightest line denoted as 0.5). Second, the value of $Risk_{Min}$ is fixed by the nature of the stock market – it is the minimum risk (by whatever metric utilized) that an asset may obtain simply by being exposed to the stock market. Third, the value of R_{max} does not change the diversification of the portfolio given that it only changes the scaling of the function. Thus, the only parameter of interest that changes our assessment of portfolio size is C_{shape} which simplifies our investigation to a single dimension.

Exhibit 2

Sampling different portions of risk versus return space leads to smaller portfolios



A family of curves corresponding to different values of C_{shape} are shown in Exhibit 2, Left (the five curves correspond to different values of Cshape – from lightest to darkest the values are 1.0, 0.5, 0.0, -0.5, and -1.0). As the value of C_{shape} decreases, it results in a change in the shape of risk versus return that allows for assets with higher return at a given risk – i.e. we are populating regions of the space in the upper left portion of the graph. Indeed, in the following section we find that when risk is defined as cash flow variance, our efficient frontier takes on a shape similar to this. We hypothesize that populating this region of higher return for a given amount of risk (i.e. decreasing C_{shape}) will result in less diversified portfolios if correlations between items are unchanged. To test this hypothesis we performed additional portfolio constructions under varying possible values of C_{shape}.

We randomly generated stocks that are bounded by different values of C_{shape} by using the R_{max} and $Risk_{min}$ values of the S&P 500. From these randomly generated stocks, we then performed mean-variance portfolio construction iteratively and asked the question: what is the average portfolio size on the efficient frontier? If our hypothesis is true, then we expect to see a reduction in the average portfolio size as the value of C_{shape} decreases regardless of the number of stocks generated. Indeed, this is what we observe as depicted in Exhibit 2, Right. There is a clear decrease of portfolio size as we approach values of C_{shape} = -1, and a marked increase until the signal attenuates at approximately a value of 1 or 2. The results shown are from portfolios constructed with 10 random sets of 50, 100, 150, and 200 stocks for a range of C_{shape} values. We clearly see a decrease in the portfolio size consistent with our hypothesis which is born out of theory independent from our first thought experiment. This result suggests that investors that maintain small portfolios must be able to identify assets with high return and low risk, otherwise they would be taking on more risk or not constructing optimal portfolios. Is it possible to identify such assets when the risk metric is changed to cash flow?

These results are due to the fact that when a particular asset has a higher expected return than any asset with higher risk, all such assets are immediately ruled out from portfolio consideration. Again, these types of assets may exist, but are difficult to identify *a priori*. The question remains: how can we identify assets that have these properties? Obviously if this were routine, it would be common practice. However, only prominent value investors seem to have tapped into the capacity to perform such analyses. We would argue that another main difference is the time horizon considered: mean-reversion to fundamental value can take on the order of years (Lakonishok, 1994) as opposed to daily/weekly/monthly returns.

How do we perform such an analysis to construct a portfolio given these results? We will demonstrate in the following sections that using fundamental analysis in the form of discounted cash flows integrated with Monte-Carlo simulations leads to a practical means of obtaining such portfolios. The integration of fundamental analysis with portfolio construction leads to exactly what we obtain when we simulate stocks in the preceding section: small portfolios.

II. Discounted Cash Flow Monte-Carlo Simulation Reveals a Distribution of Share Price Estimates

Here, we present a variation of Discounted Cash Flow (DCF) valuation integrated with a Monte-Carlo simulation to determine the variance of cash flow for all stocks in the S&P 500. Not only does this allow for direct comparison to our previous results using monthly return statistics, but it allows us to determine if the use of cash flow variance as risk will reduce the size of portfolios similar to our second thought experiment. All data for companies were scraped from the Bloomberg API (see Appendix for details and code). Specifically, we obtained revenue, operating and non-operating costs, and other value drivers in determining the free cash flow generated by companies for the past 5 years. What allows us to determine the variance of the cash flow is to resample our base case DCF by varying input parameters (described in more detail below) based on their historical variability. We note that we present only one form of a DCF but the technique is generally applicable, i.e. if one has a model that generates free cash flow and variance of input parameters, one can perform a Monte-Carlo simulation to obtain a distribution of probable free cash flows. Finally, we note that this approach is similar to one that we have utilized for over twenty years and has been published in the *Studies in Applied Finance* working paper series at the *Johns Hopkins Institute for Applied Economics, Global Health, and the Study of Business Enterprise*.

Revenue growth was assumed to be the historical average generating our top line – i.e. revenue for our 10-year forecasting period. We grow the initial revenue ($\mathbf{rev_{init}}$) at a rate determined from our analyses ($\mathbf{growth_C}$). Thus, the revenue for each forecast period **i** is:

$$Rev_i = rev_{init}(1 + growth_C)^i \qquad \textbf{(Eq. 1)}$$

where the variables are as previously defined above. From this value, we will subtract the margins to eventually determine our free cash flow.

The first step in this process is to calculate the EBITDA by subtracting all operating expenses. Suppose there are **L** operating expenses and their margins are represented by $\mathbf{margin_{OpEx,i}}$, where this margin is a function of revenue. Then, the EBITDA for each forecasting period is given by:

$$EBITDA_i = Rev_i\left(1 - \sum_{i=1}^{L} margin_{OpEx,_i}\right) \quad \textbf{(Eq. 2)}$$

where the sum on the right half of the equation is of all the operating expenses. To get to the net operating profit after interest and taxes (NOPAIT), we treat the interest expense as a margin of revenue:

$$Int_i = Rev_i * margin_{int} \quad \textbf{(Eq. 3)}$$

and taxes as a margin of EBIDTA (which in turn is a margin on revenue from Eq. 2):

$$Tax_i = EBITDA_i * margin_{tax} = Rev_i\left(1 - \sum_{i=1}^{L} margin_{OpEx,i}\right) * margin_{tax} \;. \quad \textbf{(Eq. 4)}$$

By substituting in values for EBITDA, Interest, and Taxes, we arrive at the NOPAIT:

$$NOPAIT_i = EBITDA_i - Int_i - Tax_i \quad \textbf{(Eq. 5)}$$

From here, we wish to subtract the change in working capital (**margin**$_{\Delta WC}$) and capital expenditures (**margin**$_{CAPEX}$) which will bring us to the free cash flow for the forecasting period. Since both of these are also margins of the revenue, it can be readily shown that substitution of all the values in equation 1-5 will lead to a projected free cash flow (FCF) for all periods 1-9 of:

$$FCF_i = rev_{init}(1 + growth_c)^i\left(\left(1 - \sum_{i=1}^{L} margin_{OpEx,_i}\right)(1 - margin_{tax}) - margin_{int} - margin_{\Delta WC} - margin_{CAPEX}\right) \quad \textbf{(Eq. 6)}$$

Equation 6 has important insight into how a company is valued. It is clear that the margins are an extremely important component of the cash flow. In fact, explicitly defining these margins as random normal variables in the Monte-Carlo (MC) simulation below are what lead to the variations in FCF, and hence estimated share price.

A special note should be made about the value in year 10. The terminal value year 10 is treated as a perpetuity:

$$Terminal\ Value = \frac{FCF_{10}(1 + k)}{k - growth_{LTG}} - Long\ Term\ Debt \quad \textbf{(Eq. 7)}$$

where **FCF**$_{10}$ is the cash flow in year 10, k is the discount rate we use (10%), and **growth**$_{LTG}$ is the long-term growth rate of the company (1.5%). We use a constant 10% discount rate because this represents our opportunity cost to invest in a company. We are interested in generating a 10% return on our investment (which beats the market ~ two-thirds of the time), thus we treat the free cash flow as a potential gain we may obtain through investment. This assumption and the long-term growth rate are flexible and do not change the main conclusions.

Now, we can obtain the equity value for the company by summing these cash flows that are discounted and adding back current cash and cash-equivalents:

$$Equity\ Value = \sum_{i=1}^{9} \frac{FCF_i}{(1+k)^i} + \frac{Terminal\ Value}{(1+k)^{10}} + Current\ Cash\ and\ Cash\ Equivalents$$ **(Eq. 8)**

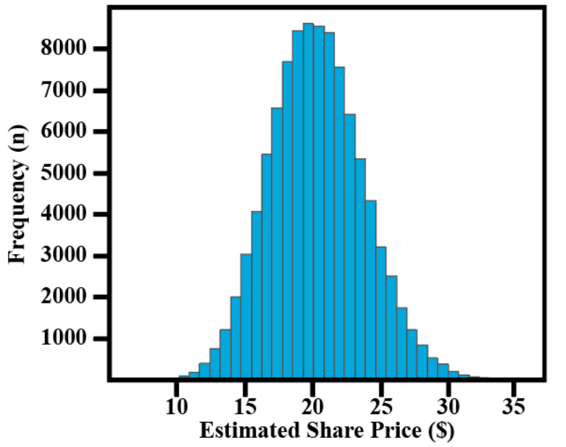
To finally arrive at the expected share price we simply divide the equity value by the number of diluted shares outstanding:

$$Share\ Price = \frac{Equity\ Value}{Diluted\ Shares\ Outstanding}$$

(Eq. 9)

Exhibit 3

DCF Monte-Carlo Simulation for NYSE: LZB



It is important to note that there are limitations to DCF analysis. For instance, the vast majority of the estimate usually comes from the terminal value, which relies on the forecasted cash flow in the future, which in turn may not be well determined. We emphasize again that this particular form of DCF and any other fundamental valuation can be used; the important point to note is that one must ascertain how well determined the input parameters are to perform in a Monte-Carlo simulation.

For value investing, we are interested in well-established companies in the S&P500, which tend to be lower growth companies that have relatively stable margins as long as they are devoid of recent large purchases and/or restructuring. As a result, the margins (i.e. expenses) will have stable well-defined distributions and standard deviations. For the purposes of this study, we treat them as normal distributions; however, this assumption can be easily tuned for any distribution. What is important is that our conclusions are not dependent on this assumption.

We use the means and standard deviations of margins and growth as input variables for a Monte-Carlo simulation, generating over 100,000 different FCF forecasts to sample the variation contained in the historical data. From the Monte-Carlo simulation, we yield a distribution of estimated share prices that are consistent with the operational history of the company (Exhibit 3). It should be noted that this represents the probable free cash flow per share distribution *given historical averages*. Since the distribution represents our uncertainty in our expected price (and thus return), it doubles as our objective measure of risk as discussed in the preceding sections (Elton (2010)). Using this methodology, we generated DCF models for every company in the S&P 500 which had reasonable statistics (see Appendix for more details). We then determined return relative to the current stock price and treated the cash flow variance as our risk metric as shown in Exhibit 3.

III. Modern Portfolio Theory as Applied to Cash Flow Variance

From the data generated in the previous section we have a measure of the distribution of probable free cash flows per share, representing our estimate of share price. Because we know the current stock price, we can readily calculate the expected return ($E[R_i]$) for each stock. We also have access to the standard deviation of free cash flows, which we will use as our risk metric as previously discussed. We also need to consider the return correlation between companies to obtain portfolio variance. To do this, we obtained returns from the last decade as described in Section 1 and found correlations between all companies to get a sense of how the stock prices will change with one another.

We now wish to take a subset of our stocks and construct a portfolio that consists of proportions ($\boldsymbol{\alpha_1, \alpha_2, \ldots, \alpha_m}$) of stocks 1 through m, and a certain proportion of cash ($\boldsymbol{\alpha_c}$). We will use cash as an asset with zero return, zero variance, and zero correlation to other stocks for simplicity. This of course is an oversimplification; however, it manifests that the entire portfolio need not consist of stocks if we are risk averse. From this construction, it follows that:

$$\sum_{i=1}^{m} \alpha_i + \alpha_c = 1 \qquad \textbf{(Eq. 10)}$$

where the summation of proportions invested in stocks and cash must equal unity by definition. We will only consider positive proportions (i.e. long positions and no shorting) and we wish to maximize the expected return while minimizing the variance of the portfolio similar to other "mean-variance" optimization processes. To accomplish this, we utilized modern portfolio theory and minimized the variance across all possible expected returns (Francis and Kim (2013)).

Exhibit 4

A Novel Risk Metric - Modern Portfolio Theory Optimization on DCF Monte-Carlo Simulations of S&P 500

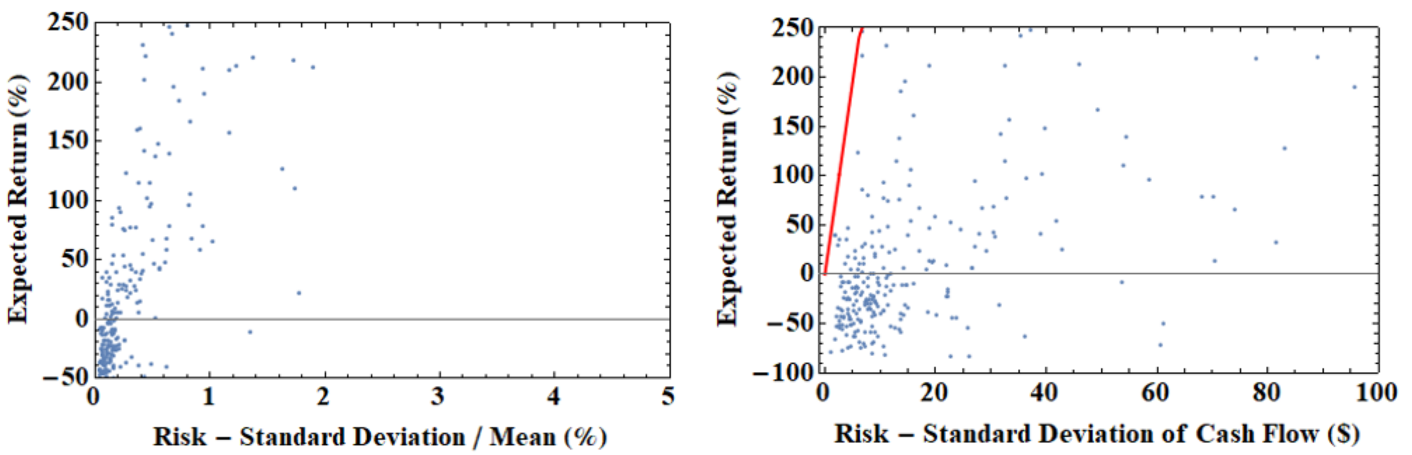
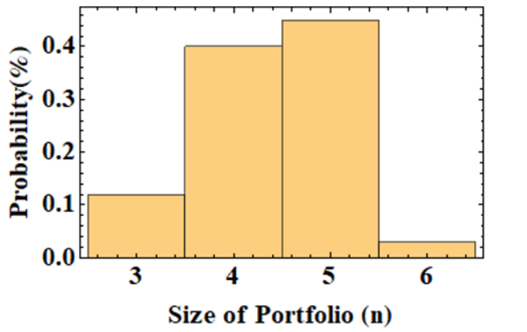


Exhibit 4 shows two plots resulting from the analysis above. The plot on the left shows the expected return versus risk as a percentage similar to plots we have shown and are found in the literature. The risk is measured by the coefficient of variation – i.e. the uncertainty of the stock price (standard deviation) relative to the current stock price resulting in a percentage. This plot is the standard presentation form in the literature – i.e. the risk metric is the percent standard deviation (similar to Exhibit 2), and in the case of modern portfolio theory, the temporal fluctuation of a stock price. However, an individual investor or asset manager does not necessarily think in terms of these values. Furthermore, given access to the risk metric of nominal dollars from our Monte-Carlo simulation, we can recast the risk metric as the standard deviation of cash flow per share (i.e. our share price estimate).

This results in the figure on the right of Exhibit 4 showing the return versus risk for every company along with the efficient frontier in red. Two main conclusions can be drawn. First, the DCF-MC analysis yields a wide variety of returns and standard deviations (risk) for the various companies. Second, the efficient frontier appears to have very little curvature and almost vertical. This is indeed the shape that we would expect to observe for portfolios with few items as demonstrated by our simulations in Exhibit 2 (i.e. a low C_{shape}). The importance of this figure cannot be overstated - these results lead us to conclude that we have constructed portfolios that are optimal according to MPT using our new risk metric.

Exhibit 5

Novel Risk Metric Results in Smaller Optimal Portfolios



Since the shape of the efficient frontier suggests a low C_{shape}, we would expect the efficient portfolios to consist of a smaller number stocks as demonstrated in the first section. This is the exact result we find – i.e. the majority of portfolios are small and contain five or fewer securities as shown in Exhibit 5. The conclusion is clear: most, if not all, of the portfolios on the efficient frontier consist of few securities in apparent contradiction of what is generally expected for efficient portfolios. Indeed, we have reduced the average number of items in the portfolio from ~17 to ~5 by using cash flow variance as our risk metric. Specifically, this approach has yielded portfolio sizes that average 4.4 ± 0.7 (mean ± standard deviation).

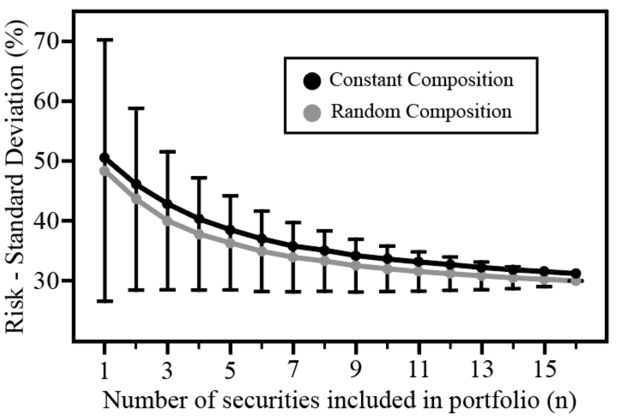
Although the results in Exhibit 5 are consistent with empirical observations of "small is beautiful" from practitioners as mentioned in the introduction, and exactly the results we would expect from a low C_{shape}, they are still in apparent contradiction to previous work that found approximately twenty to thirty securities are required to diversify away unsystematic risk (Evans and Archer (1968); Statman (1987); Statman (2004)). We address this issue with two further analyses.

First, we note that our approach has yielded risk values for certain companies in Exhibit 4 that are lower in absolute and relative magnitude than those observed in Exhibit 2. Given that both approaches have cash as an asset with zero expected return and zero correlation to other stocks, this may result in smaller portfolios if you have access to lower risk stocks. To control for this issue, we repeated the analyses in Exhibit 4 by arbitrarily increasing the risk of all stocks and determining how this changed the average size of the portfolio. We did observe a small effect, however, the effect attenuates rapidly with increased risk and still results in portfolios that are significantly smaller than those in Exhibit 2. For instance, when we increase the risk of the entire universe of stocks by \$10 (which results in a similar plot to Exhibit 2), we still end up with an average optimal portfolio size of ~7 stocks. Furthermore, when we increase the risk by \$50 which is now significantly deviant from reality (i.e. now a 95% confidence interval of our share price estimate is ± \$100), we still yield an average optimal portfolio size of ~8 stocks. This leads us to conclude that small portfolios are not an artifact of small risk in the form of low cash flow variance in a company forecast. However, we should note that this may be the case with an investor – e.g. they could be quite confident in their subjective valuation of a company. Thus, we believe the plot in Exhibit 4 is still realistically reflective of the decision-making process an investor faces.

Second, we wanted to control for the DCF Monte-Carlo approach that yielded our estimates of return and variance (see Section II). Thus, we repeated simulations from previous studies that investigated optimal portfolio size (Evans and Archer (1968)). We hypothesized that although the *average* size of optimal portfolios might be higher than our results, if we were to take into account the *distribution* of optimal portfolios, it is still possible to arrive at small optimal portfolios. Akin to previous analyses, we randomly picked stocks, weighed them equally in a portfolio, and then determined the portfolio risk by standard deviation of daily returns. We also performed this analysis by randomly weighting the stocks to see how much composition plays a role in overall portfolio risk.

Exhibit 6

Risk is attenuated upon addition of a few securities



The results from this analysis are shown in Exhibit 6 with constant composition portfolios in black and random composition in gray. The plot shows the risk as measured in standard deviation (%) versus the number of securities included in the portfolio. The attenuation of risk is used as a means of determining the required number of securities in a portfolio in order to diversify unsystematic risk. We note that both methods result in a monotonically decreasing *average* as depicted by the dots. However, unlike previous analyses, we have access to the *distribution* of risk values as shown by the error bars in black in Exhibit 6 (gray not shown for simplicity of the figure; the same conclusions apply). This result further demonstrates that: (1) there is large variability in the risk (standard deviation) of the portfolios generated at small sizes, and (2) that a few securities is still enough to generate a portfolio that has low risk.

Taken together, our results in Exhibits 4 and 5 coupled with our control analyses lead us to conclude that the origin of small portfolios is an artifact of low risk stocks or the DCF Monte-Carlo approach. Thus, we conclude that the likely origin of these results is the novel risk metric being employed: cash flow standard deviation in nominal dollars. Finally, although there is substantial empirical evidence for small portfolios in value investment funds, private-equity firms, and hedge funds, this represents (to our knowledge) the first quantitative formalism to reach such a conclusion – i.e. “small is beautiful.”

Implications of Integrating Fundamental Analysis with Portfolio Design

Taken together, the results of the previous sections suggest that if fundamental analysis for value investing is integrated with portfolio construction using a different objective risk metric, it is possible to diversify away unsystematic risk with a smaller number of securities than previously suggested. What is clear from Exhibit 6 is that this result is not an artifact from limiting our selection of securities, and that the conclusions we obtain from our fundamental analysis are robust.

Three unique aspects of our approach reveal the true power of this methodology in portfolio construction from an academic and investor perspective. The first is that the fundamental analysis DCF using a MC simulation ascertains how well determined the forecasted free cash flows per share is for our companies of interest. Indeed, the question we are able to answer from such an analysis is: if the company operates within its historical averages, what should the expected free cash flow distribution per share look today? This can be unequivocally determined from our analysis and allows us to ensure that an investor is unlikely to lose money if the company is significantly undervalued. This is what allows us to populate the top left portion of the return/risk space as discussed in the first section. The variability in the distribution is an excellent metric for risk, representing the uncertainty in the company's operating behavior. Of course this does not rule out the possibility of bankruptcy or mismanagement of the company, but as Markowitz points out, this is where research on a company, subjectivity, and the, "judgement of practical men," will come into play (Markowitz (1952)).

The second unique aspect is that we are able to directly take into account the price of the securities in question. Having objectively identified the probable distribution of the stock price, we are able to directly compare this value to the known price in the market to determine the expected return distribution. Even as pointed out by Markowitz, MPT focuses merely on how to optimize composition only once the expected return statistics are known. Indeed, this new method allows an investor to directly pose the question: is the security over or undervalued in an absolute dollar sense?

The third unique aspect of our approach is that we can integrate the statistics from the DCF-MC directly into portfolio composition. This leads us to conclude that the optimal and efficient way to construct a portfolio is by concentrating investments in a few securities that are unlikely to lose by fundamental analysis. What is truly remarkable is that this investment strategy is already employed in the industry: specifically by value investors, private-equity firms, and hedge-funds. In reality, this result should not come as too surprising since resampling theory has demonstrated that ~10 to ~20 samples may be enough to determine robust statistics (Hanke and Mehrez (1979)). The work presented here sheds light into the mechanism of why small is beautiful: fundamental analysis leads you to invest in companies where it is unlikely to lose money, and as a result, the amount of securities you need to hold to diversify away risk decreases. This leads to portfolios that are "spring-loaded" for a big winner without dampening the overall return with over-diversification.

References:

Aliber, Robert Z., 2011. *Your Money and Your Life: A Lifetime Approach to Money Management* (Stanford Univeristy Press, Stanford, California).

Brealey, Richard A., Stewart C. Myers, and Alan J. Marcus, 2015. *Fundamentals of corporate finance* (McGraw-Hill/Irwin, New York, NY).

Campbell, J. Y., 1991, A variance decomposition for stock returns, *The Economic Journal* 101, 157-179.

Elton, E. J., and M. J. Gruber, 1977, Risk reduction and portfolio size: An analytical solution, *The Journal of Business* 50, 415-37.

Elton, Edwin J., 2010. *Modern portfolio theory and investment analysis* (J. Wiley & Sons, Hoboken, NJ).

Evans, J. L., and S. H. Archer, 1968, Diversification and the reduction of dispersion: An empirical analysis, *The Journal of Finance* 23, 761-67.

Fama, E. F., 1965, The behavior of stock-market prices, *The Journal of Business* 38, 34-105.

Fama, E. F., and K French, R., 1988, Permanent and temporary components of stock prices, *Journal of Political Economy* 96, 246-273.

Fama, E. F., and K French, R., 1992, The cross-section of expected stock returns, *The Journal of Finance* 47, 427-465.

Fama, E. F., and K French, R., 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fisher, I., 1930. *The theory of interest* (The Macmillan Company, New York).

Fisher, Irving, 1922, *The Making of Index Numbers: A Study of Their Varieties, Tests, and Reliability* (Houghton Mifflin Company, Boston).

Francis, Jack Clark, and Dongcheol Kim, 2013. *Modern portfolio theory : Foundations, analysis, and new developments + website* (John Wiley & Sons, Hoboken, N.J.).

Goetzmann, William N., 2016, *Money Changes Everything: How Finance Made Civilization Possible* (Princeton University Press, Princeton, N.J.).

Graham, B., 1934. *Security analysis* (McGraw-Hill, New York).

Hanke, S. H., and A. Mehrez, 1979, An optimal sampling procedure for the collection of residential water use data, *Water Resources Research* 15, 1343-1348.

Kaufman, Peter D., 2008. *Poor Charlie's Almanack,* Expanded third Edition (PCA Publication, L.L.C., Virginia beach, Virginia).

Lakonishok, J., A. Schleifer, and R. W. Vishny, 1994, Contrarian investment, extrapolation, and risk, *The Journal of Finance* 49, 1541-1578.

Lo, A. W., 2010. *Hedge funds: An analytic perspective* (Princeton University Press).

Lowe, Janet, 1997. *Warren Buffett Speaks: Wit and Wisdom from the World's Greatest Investor* (John Wiley and Sons, Inc., New York).

Mandelbrot, B., 1963, The variation of certain speculative prices, *The Journal of Business* 36, 394-419.

Markowitz, H., 1952, Portfolio selection, *The Journal of Finance* 7, 77-91.

Markowitz, H., 1999, The early history of portfolio theory: 1600-1960, *Financial Analysts Journal* 5-16.

Poterba, J. M., and L. H. Summers, 1988, Mean reversion in stock prices: Evidence and implications, *Journal of Financial Economics* 22, 27-59.

Roy, A. D., 1952, Safety first and the holding of assets, *Econometrica* 20, 431-449.

Rubinstein, M., 2002, Markowitz's "portfolio selection": A fifty-year retrospective, *The Journal of Finance* 57, 1041-45.

Shakespeare, William, 1967. *The merchant of venice; a comedy* (Stanwix House, Pittsburgh,).

Sharpe, W F., 1963, A simplified model for portfolio analysis, *Management Science* 9, 277-93.

Sharpe, W F., 1964, Captial asset prices: A theory of market equilibrium under conditions of risk, *The Journal of Finance* 19, 425-442.

Sharpe, W F., 1970. *Portfolio theory and capital markets* (McGraw-Hill, New York).

Sharpe, W F., 1981. *Investments*.

Sindreu, John., 4-5 January 2020. "Go AHead, Put a Lot of Eggs in ths Basket," *Wall Street Journal*

Statman, M., 1987, How many stocks make a diversified portfolio, *Journal of Financial and Quantitative Analysis* 22, 353-363.

Statman, M., 2004, The diversification puzzle, *Financial Analysts Journal* 60, 44-53.

Wigglesworth, Robin, 4-5 January 2020. "Stockpickers pray for turnaround as passive exodus marches oon," *Financial Times.*

Appendix I: Description of the Discounted Cash Flow Monte-Carlo

Ticker names for the S&P500 were obtained from Wikipedia using *Python* and the *BeautifulSoup* package. These tickers were then saved into a spreadsheet and used to obtain values required for the DCF-MC. Specifically, the following functions were used on a Bloomberg Terminal to interface with the API:

Value	Bloomberg API Code
Revenue	*SALES_REV_TURN*
Cost of Revenue	*IS_COGS_TO_FE_AND_PP_AND_G*
Depreciation & Amortization	*CF_DEPR_AMORT*
Operating Expenses	*IS_OPERATING_EXPN*
Adjusted Non-Operating (Income) Loss	*IS_NON-OPERATING_INCOME_LOSS*
Adjusted Net Interest Expense	*IS_ADJUSTED_NET_INTEREST_EXPENSE*
Income Tax Expense	*IS_INC_TAX_EXP*
Cash & Cash Equivalents	*BS_CASH_NEAR_CASH_ITEM*
Total Shares Outstanding	*EQY_SH_OUT*
Long-Term Debt	*BS_LT_BORROW*

We obtained five years of historical data from 2010 through 2014 for all fields listed above except for Cash & Cash Equivalents, Total Shares Outstanding, and Long-Term debt for which we obtained the year-end values for 2014. Thus, we setup our simulation as though we have access to all data from 2014 and are picking stocks as of approximately the end of the first quarter of 2015 when these SEC filings would be available.

To perform the DCF-MC for each stock, the data above were read into a *Python* script (see Appendix II: Code for more) and stored in a data frame using the *Pandas* package. Subsequently, these historical data were used to perform 100,000 trials of a Monte-Carlo simulation as described in the main text. Briefly, historical statistics were calculated for all input variables of interest, and a random 10-year DCF was randomly generated using these statistics. To save memory, only the ticker, percentile of the current stock price, current stock price from *Yahoo Finance* as of 3/3/2015, mean of the MC, and standard deviation of the MC were saved in an output file. These were the data that were directly utilized to perform portfolio optimization that resulted in Exhibits 4 and 5.

Appendix II: Code

The following code for *Screen_SP500_MC_DCF.py* was used to read in the raw Bloomberg API data and perform the Monte-Carlo simulation.

```
"""
Name:         Screen_SP500_MC_DCF.py
By:           Hesam N. Motlagh, PhD
Created:      2015.03.03
Last Updated: 2020.03.22
--------------------------------------------------------------------------------
This script was written for our modern portfolio theory research paper.
The code reads in the raw data from the Bloomberg API and then performs a
100,000 trial MC simulation for all stocks.
"""
#import the classes of interest
import pandas as pd
import numpy as np
from yahoo_finance import Share
import scipy.stats
from scipy import stats
"""
--------------------------------------------------------------------------------
START OF FUNCTIONS FOR PERFORMING THE MONTE-CARLO DCF SIMULATION BRO!
--------------------------------------------------------------------------------
"""
#The following function calculates the YoY revenue growth
def rev_growth(rev):
    changes = []
    for idx in range(len(rev)-1):
        changes.append((rev[idx+1]-rev[idx])/rev[idx+1])
    return changes
#the following function calculates margins
def get_margin(big,small):
    margins = []
    for idx in range(len(big)):
        margins.append(small[idx]/big[idx])
    return margins
#this function performs a single trial of the MC simulation
def get_dcf_val(rev_i,ltd,cso,rev_change,cogs_da_marg,opex_marg,nonopex_marg,\
                taxmarg,intmarg,cash,capexmarg):
    #start calculating parameters of interest
    rev_change = np.array(rev_change)
    rev_vals = [rev_i]
    #perform random revenue generation
    for idx in range(9):
        rev_vals.append(rev_vals[idx]*(1+np.random.normal(rev_change.mean(),\
                        rev_change.std())))
    #now let's get EBITDA!
    cogs_da_marg = np.array(cogs_da_marg)
```

```
    opex_marg = np.array(opex_marg)
    nonopex_marg = np.array(nonopex_marg)
    ebitda = []
    for idx in range(len(rev_vals)):
        if(cogs_da_marg.std() == 0.0):
            marg1 = cogs_da_marg.mean()
        else:
            marg1 = np.random.normal(cogs_da_marg.mean(),cogs_da_marg.std())

        if(opex_marg.std() == 0.0):
            marg2 = opex_marg.mean()
        else:
            marg2 = np.random.normal(opex_marg.mean(),opex_marg.std())

        if(nonopex_marg.std() == 0.0):
            marg3 = nonopex_marg.mean()
        else:
            marg3 = np.random.normal(nonopex_marg.mean(),nonopex_marg.std())

        ebitda.append(rev_vals[idx]*(1-marg1-marg2-marg3))
    #ok, let's determine the tax expense
    taxmarg = np.array(taxmarg)
    taxes = []
    for idx in range(len(ebitda)):
        try:
            taxes.append(ebitda[idx]*np.random.normal(taxmarg.mean(),\
                             taxmarg.std()))
        except ValueError:
            taxes.append(0.0)
    #Now just subtract interest (except period 10)
    interest = []
    intmarg = np.array(intmarg)
    for idx in range(len(rev_vals)-1):
        try:
            interest.append(rev_vals[idx]*np.random.normal(intmarg.mean()\
                                ,intmarg.std()))
        except ValueError:
            interest.append(0.0)
    interest.append(0.0)
    #Now just subtract wc/capex and we've arrived @ the FCF estimate
    capex = []
    capexmarg = np.array(capexmarg)
    for idx in range(len(rev_vals)):
        capex.append(rev_vals[idx]*np.random.normal(capexmarg.mean(),\
                         capexmarg.std()))
```

```
    #ok, let's calculate the FCF for period one through 10 now
    fcf = []
    for idx in range(len(ebitda)):
        fcf.append(ebitda[idx]-taxes[idx]-interest[idx])#-dWC[idx])#-capex[idx])
    #alright, now let's calculate the DCF for period one through nine
    dcf = []
    #print fcf
    for idx in range(len(fcf)-1):
        dcf.append(fcf[idx]/(1.1**(idx+1)))
    #ok, now let's calculate the terminal value
    tv = ((fcf[9]*(1.1)/(0.1-1.5/100.0)-ltd)/(1.1**10))
    dcf.append(tv)
    return (sum(dcf)+cash)/float(cso)

"""
--------------------------------------------------------------------------------
START OF THE CODE TO RUN THIS FOR THE ENTIRE UNIVERSE OF STOCKS
--------------------------------------------------------------------------------
"""
#ok, let's read in the raw data taken from the bloomberg terminal API
#using Excel
temp = open('SP500_data.csv','r')
lines = temp.readlines()
temp.close
#let's extract the tickers and the raw data that I will use for the MC-DCF
data, tickers = [], []
for line in lines[2:len(lines)]:
    test = line.strip("\n").split(",")
    try:
        to_add = []
        for x in test[1:len(test)]:
            to_add.append(float(x))
        data.append(to_add)
        tickers.append(test[0].split()[0])
    except ValueError:
        print(test[0].split()[0] + " Had some data removed")


#extract what the column titles should be
valstemp, yrs = lines[0].strip("\n").split(','), lines[1].strip("\n").split(',')
prev_idx, clmns = 0, []
for idx in range(len(valstemp)):
    if(valstemp[idx] == ''):
        clmns.append(yrs[idx] + " " + valstemp[prev_idx])
    else:
```

```
        prev_idx = idx
        clmns.append(yrs[idx] + " " + valstemp[idx])
#alright, let's make a data frame
clmns = clmns[1:len(clmns)]
df = pd.DataFrame(data,index=tickers,columns=clmns)
df[clmns] = df[clmns].astype('float32')
#ok, let's make an output file
outfilename = 'Aut_DCF_SP500_n100k.dat'
output = open(outfilename,'w')
output.close()
#Let's loop over every ticker and perform a HG-DCF
for tick in tickers:
    print("working on " + tick)
    years = ['2010','2011','2012','2013','2014']
    dat = df.loc[tick]
    #first get the revenue growth
    rev_vals = []
    for yr in years:
        rev_vals.append(dat[yr + " Revenue"])
    rev_change = rev_growth(rev_vals)
    #calcuate the cogs_net_da margins and CAPEX = D&A
    cogs, da = [], []
    for yr in years:
        cogs.append(dat[yr + " Cost of Revenue"])
        da.append(dat[yr + " Depreciation & Amortization"])
    cogs_netda = [cogs[idx]-da[idx] for idx in range(len(cogs))]
    cogs_da_marg = get_margin(rev_vals,cogs_netda)
    capexmarg = get_margin(rev_vals,da)
    #ok, now we want to calculate the other operating expenses
    opex = []
    for yr in years:
        opex.append(dat[yr + " Operating Expenses"])
    opex_marg = get_margin(rev_vals,opex)
    #now get non-operating expenses
    nonop_exp = []
    for yr in years:
        nonop_exp.append(dat[yr + " Adjusted Non Operating (Income)/Loss"])
    nonopex_marg = get_margin(rev_vals,nonop_exp)
    #now we can get the EBITDA margins
    #now we have the EBITDA values!
    ebitda = [(1.0-cogs_da_marg[idx]-opex_marg[idx]-nonopex_marg[idx])*rev_vals[idx]\
            for idx in range(len(cogs))]
    #ok, now calculate the income taxes as a function of ebitda
    taxes = []
    for yr in years:
```

```
    for yr in years:
        taxes.append(dat[yr + " Income Tax Expense"])
    taxmarg = get_margin(ebitda,taxes)
    #ok, now calculate the interest expense
    intexp = []
    for yr in years:
        intexp.append(dat[yr + " Adjusted Net Interest Expense"])
    intmarg = get_margin(rev_vals,taxes)
    #ok, now we are ready to perform a dcf with these values
    ltd = dat['2014 Long_Term Debt']
    revi = dat['2014 Revenue']
    cso = dat['2014 Current Shares Outstanding']
    cash = dat['2014 Cash & Near Cash']
    #Let's run 100,000 trials of hte MC-DCF
    mc_vals = []
    for idx in range(100000):
        mc_vals.append(get_dcf_val(revi,ltd,cso,rev_change,cogs_da_marg,\
                          opex_marg,nonopex_marg,taxmarg,intmarg,cash,capexmarg))
    #ok, let's get the current price and other data
    share = Share(tick)
    curr_price = float(share.get_price())
    percentile = stats.percentileofscore(mc_vals,curr_price)
    mc_vals = np.array(mc_vals)
    #finally let's print that we made it this far and output the data
    print(tick + "\t" + str(percentile))
    output = open(outfilename,'a')
    to_write = tick + "," + str(percentile) +","+ str(curr_price) +","+\
               str(mc_vals.mean())+","+str(mc_vals.std())
    output.write(to_write+"\n")
    output.close()
    print("Done!")
```