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EQUITY RELEASE: ANOTHER EQUITABLE IN THE MAKING

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Johns Hopkins Institute for Applied Economics, Global Health, and the Study of Business Enterprise

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About the Series

The *Studies in Applied Economics* series is under the general direction of Professor Steve H. Hanke, co-director of The Johns Hopkins Institute for Applied Economics, Global Health, and the Study of Business Enterprise.

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Abstract

A major problem emerging in the UK equity release sector is the undervaluation by firms of their No-Negative Equity Guarantees (NNEGs), which cap borrowers' repayments to the maximum of the rolled-up loan value and the value of their property at the time of repayment. This undervaluation arises from the common use by firms of an incorrect valuation methodology, the Discounted Projection or Real World approach, in which valuation is based on projected future house prices. The correct, Market Consistent (MC), approach uses forward house prices as the underlying variable in the relevant put option pricing equation. Results indicate that the MC approach produces much higher NNEG valuations and suggest that firms have considerably undervalued their NNEGs. Our analysis reinforces recent analysis by the UK Prudential Regulation Authority and especially their Good Practice Principles for the management of equity release mortgages. This NNEG undervaluation story bears a number of similarities to the Equitable Life fiasco of almost two decades ago and suggests that some of the problems present in the Equitable saga – especially the undervaluation of opaque long-term guarantees and the undermining of actuarial practice by commercial interests – have still to be resolved.

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JEL Codes: G22, G32, G38

In the Equity Release Council's Spring 2018 *Market Report*, its chairman David Burrowes struck a reassuring tone:

> Annual lending activity by our members has surpassed £3 billion for the first time and customer numbers reached 67,000 in 2017. Property wealth is increasingly recognised by people as a safe and sought-after source of retirement finance, with the market attracting twice as many new customers as it was five years ago. …
>
> The range of product options available to equity release customers has grown 25% year-on-year, providing more choice to underpin a robust and competitive market.
>
> Looking forward, we expect the need for new sources of income in retirement will continue to grow as many people will be unable to rely on pressured pension pots. (Equity Release Council, 2018, p. 2)

Mr. Burrowes omitted however to mention a fly in the ointment that had been causing whispers among the Equity Release Mortgage (ERM) in-crowd for a little while now. The problem is that some firms are under-valuing the No Negative Equity Guarantees (NNEGs) that are a standard feature of most ERM products. This under-estimation seems to be on a large scale too.

These concerns received some publicity with the publication on 7 August this year of reports by BBC business journalist Howard Mustoe (Mustoe, 2018) and the Adam Smith Institute (Dowd, 2018b) on the issue, and with the airing that evening of a BBC Radio 4 programme, "The Equity Release Trap."[1]

This story is a complicated one and its roots go back some time. A couple of decades ago, there was a scandal surrounding Equitable Life, which had been under-valuing opaque and apparently innocuous long-term guarantees since the 1950s. Equitable came to grief in 2000 when it was no longer able to keep its promises. There was then a big outcry and the insurance regulatory system was overhauled to make sure that an Equitable-style fiasco never happened again.

It would appear that that overhaul hasn't worked.

In both cases, there was a toxic combination of intellectual error and short-term thinking. In the Equitable case, there was an underlying presumption that the guarantees in question, Equitable's Guaranteed Annuity Rate (GAR) options, didn't really matter and that any problems that they might entail were well into the future anyway. In the equity release case, the intellectual error involves a profound misunderstanding of option pricing theory by professional actuaries, combined with a mindset on the part of industry leaders that puts short-term profitability and 'competitiveness' ahead of notions of long-term sustainability. When it comes to NNEG valuation, this mindset prioritises *low* NNEG valuations over *sound* NNEG valuations and the rest is obvious.

[1] Since then, there has been considerable public discussion of the NNEG valuation issue. We provide a commentary on our blog, The Eumaeus Project (eumaeus.org/wordp/).

The intellectual error centres around the underlying variable in the option pricing formula. In the case of vanilla Black-Scholes (BS), the underlying might typically be a stock and the current stock price would be the first term entered into the BS put option pricing equation. The NNEG situation is a little more complicated. To start with, a NNEG involves a portfolio of put options and we are dealing with puts on forward contracts rather than options on spot underlyings. For example, if a customer takes out an ERM at the age of 70, there is a NNEG put for the possibility that the ERM loan might end when the customer is 71, another NNEG put for the possibility that the ERM loan might end when the customer is 72 and so forth. Each of these put options is issued now, but has a horizon of one, two, etc. years in the future. The price that enters into each put option pricing equation is not the spot price of the underlying, but the forward price of the underlying, and the underlying itself is not a stock but a house. So for the put option that ends in future year t, the underlying is the forward house price for year t, the price agreed now for a house to be delivered and paid for in year t. This approach is based on standard option-pricing theory as per Black (1976). In actuarial circles, this approach (or something close to it) is sometimes called the 'Market Consistent' approach to NNEG valuation.

The problem is that a number of practising actuaries in the UK equity release sector have convinced themselves that the underlying price that is relevant for put option pricing is not the *forward* house price for year t but the *future* house price or expected future price for year t. However, forward and future prices are *very different* and to confuse the two is to commit a major logical error. This error is a *big deal* because inputting the expected future house price into the option-pricing equation gives very low NNEG valuations, whereas inputting forward house prices into it gives much larger NNEG valuations. This second, incorrect, approach is commonly referred to in actuarial circles as the "Real World" or "Discounted Projection" approach.

A difference however between the Equitable Life and equity release cases is that when Equitable started issuing GARs in the 1950s, the valuation of options was not well-understood. The option pricing breakthrough only occurred in 1973 with the publication of the famous articles by Black, Scholes and Merton (Black and Scholes, 1973; Merton, 1973). The early (though not later) misvaluations of GARs by Equitable Life are then to some extent excusable. There is no such excuse with equity release, however. Both the principles and the nuances of option valuation have been well known for decades and are taught in universities all over the world.

It is curious, too, that the UK actuarial professional association, the Institute and Faculty of Actuaries (IFoA), has yet to speak out against these unsound NNEG valuation practices. It is also on the record as endorsing a number of misconceptions on NNEG valuation.

Welcome to Equitable 2.0.

This article is organised as follows. The next section explains the basic economics of Equity Release. The second section looks at the timing and expected timing probabilities of the customer exiting the house. The third explains the issues involved in valuing ERM products and their NNEG guarantees. The fourth gives an example valuation. The fifth examines the sensitivity of valuations to key input assumptions. The sixth section presents the results of some stress tests. The seventh examines the UK's Prudential Regulation Authority's principles of ERM good practice and their implications for ERM and NNEG valuation. The eighth deconstructs some actuarial misconceptions about these principles. The ninth looks at the apparently widespread use of the indefensible "Real World"/"Discounted Projection" approach in the UK equity release sector. The tenth section delves into the origins of this approach. The eleventh section examines the impact of net rental rate assumptions on NNEG valuations. The twelfth section revisits the lessons learned (or not) in the aftermath of the Equitable case and the final section concludes. The article is followed by four appendices elaborating on issues that arise in the course of the discussion: the calibration of the net rental rate, why deferment values are lower than current property values, the use by Just Group of a 4.25% expected house price inflation rate in the valuation of its NNEGs, and whether the NNEG valuation approaches currently used by UK equity release firms meet the Financial Reporting Council's Technical Actuarial Standards (TASs).

The Home Economics of Equity Release

Explanation and mechanics

An ERM (sometimes also known as a Lifetime Mortgage) is a type of loan collateralised by a property ('house'), and the particular class of ERM we are interested in goes as follows.[2] The loan is taken out by a customer late in life who owns the property they live in. The customer uses the loan to supplement their income, help their children get on the property ladder or whatever. Unlike a normal loan, this loan has no fixed end date and involves no regular interest payments. Instead, the loan ends when the customer exits the house, either by death or by going into a nursing home, and the amount owed on the loan accumulates over time until the loan is repaid.[3] At the time of exit, the lender takes possession of the property and sells it to repay the loan. If there are any proceeds, these are returned to the customer or to their estate.

The ERM loan will be taken out as some fairly low proportion of the property value – 40% is typical for a 70 year old, but Loan to Value ratios (LTVs) tend to be lower for lower ages and higher for higher ones – and the lender is protected against any risk of loss for as long as the loan value is below the value of the house.

The loan rate will be fixed at the inception of the loan and current new loan rates are about 6%.

The value of the collateral, the house, will vary with the house's market price. Typically, house prices have risen in recent years and we might (or might not!) expect them to continue to rise, but we would not usually expect the house price to rise at a rate

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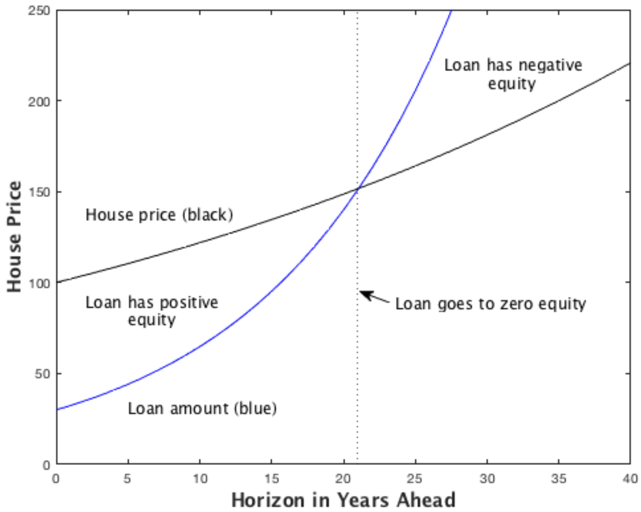
[2] We are not concerned here with other types of equity release product such home reversions, in which the borrower sells all or part of their property at less than its market value in return for a tax-free lump sum, a regular income, or both, but stays on in their home as a tenant who pays no rent. Nor are we concerned with ERM loans that do not incorporate NNEGs, suffice to note that most incorporate NNEGs and that all ERMs issued by members of the Equity Release Council do.

[3] In some cases, the loan can also end by early repayment, but we do not consider early repayment or prepayment risk. A more sophisticated analysis would certainly do so, but any treatment of this risk factor would be a refinement to our analysis and be unlikely to lead to substantially different conclusions.

exceeding the loan rate. In any case, house prices are uncertain and sometimes fall, so expectations of future house prices are unlikely to be exactly realised.

A typical case is shown in Figure 1.

Figure 1: Loan Equity in a Typical Equity Release Mortgage



In this case, the initial house price is £100 and the initial value of the loan is £30, so the Loan-to-Value (LTV) is 30%. Over time we expect both the loan amount (shown in blue) and the house price (in black) to rise, but the loan amount will rise at a faster rate and eventually, if the customer lives long enough, the blue loan amount line will cross over the black house price line. Thereafter the loan amount will exceed the value of the house, i.e., the loan will go into negative equity.

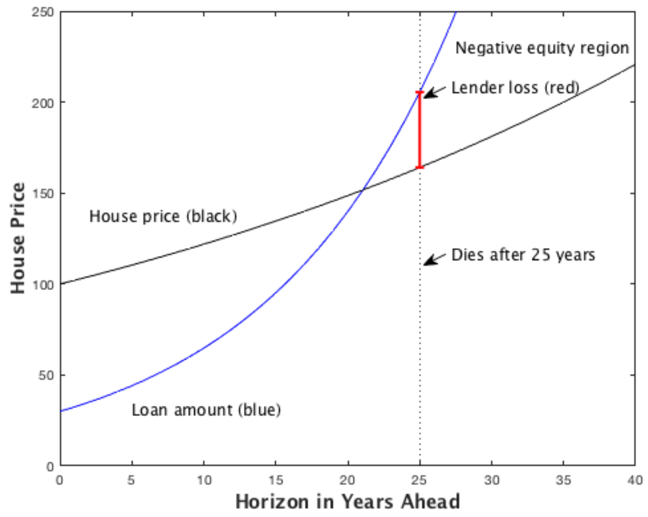
If the customer exits the house before the point of negative equity (e.g., 21 years in Figure 1), then the lender would be repaid in full.

If the customer exits after that point, the loan would expire in negative equity, i.e., the value of the property collateral would not be enough to cover the accumulated loan amount. In the absence of a NNEG, the lender could sue the borrower or their estate, but there might have few assets left, especially if the borrower was moving into a retirement home and any remaining assets were being used to finance their care. Most ERMs incorporate a NNEG, however, and in such cases the negative equity becomes a loss borne by the lender.

Another way to think about the ERM-with-NNEG contract is that it gives the lender the minimum of the house price (black) and loan amount (blue) lines. The fact that the lender gets the minimum of two values indicates that the contract involves the lender granting a put option to the borrower.

The lender's potential loss with the NNEG in place is illustrated in Figure 2, and let's henceforth assume for convenience that exit is due solely to death:

Figure 2: ERM Loan Expires in Negative Equity

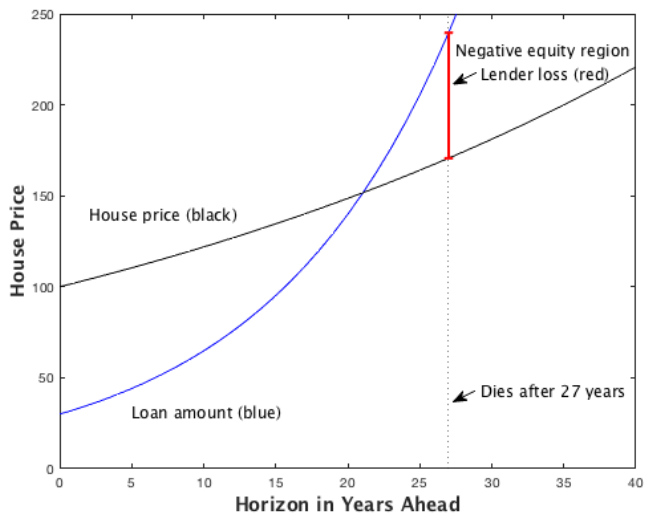


In this case, the borrower dies after 25 years and the lender makes the loss given in red, the difference between the loan value and the house price after 25 years.

We should recognise that this loss (and whether any loss occurs at all) is uncertain before the event. The timing of death is uncertain and if the customer dies early then there would be no loss to the lender. But if the customer dies later the lender suffers a loss that depends among other factors on the timing of death.

So if the customer dies after 27 years, then the dotted time-of-death line in Figure 2 would be moved 2 years to the right and the loss would be larger than in Figure 2. This case is shown in Figure 3:

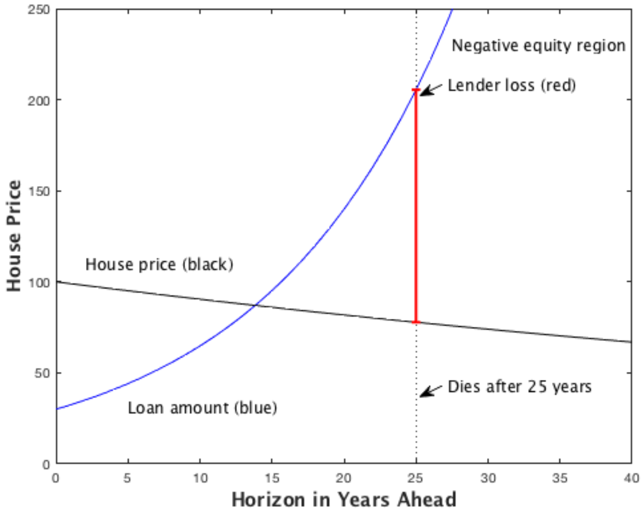
Figure 3: Time of Death and Lender Loss



Thus, the NNEG potentially exposes ERMs to longevity risk – the risk that the customer might live too long.

ERMs are also exposed to house price risk. This risk is illustrated in Figure 4:

Figure 4: The Impact of a Fall in House Prices on Negative Equity



The house price might be lower at the time of death than the lender expected it to be. Figure 4 shows a case where the house price declines instead of rising. If the customer dies after 25 years, then it is clear from a comparison of Figures 2 and 4 that the lender suffers a bigger loss due to the house price fall. ERMs are therefore subject to house price as well as longevity risk.

ERMs are also subject to a number of other risk factors. These include, e.g., the risk-free interest rate and the volatility of the (forward) house price.

Economic considerations

From the borrower's perspective, taking out an ERM loan might be a suitable choice for an older individual or couple who are asset rich but cash poor, e.g., they might have a need for cash or wish for a higher standard of living in retirement. One can also imagine additional circumstances in which an ERM might be suitable, e.g., because their children may be affluent or because they don't want to leave their children any inheritance, or because they may have no children and don't want to leave their house to a cats' home. For such people, a regular mortgage would not normally be practical because they would no longer be working and therefore not have the income to repay such a mortgage.

From the lender's perspective, an ERM loan offers a high loan rate and is highly collateralised, at least to start with. Its main downside is the impact of the NNEG, which is the core focus of this report.[4]

Exit Probabilities

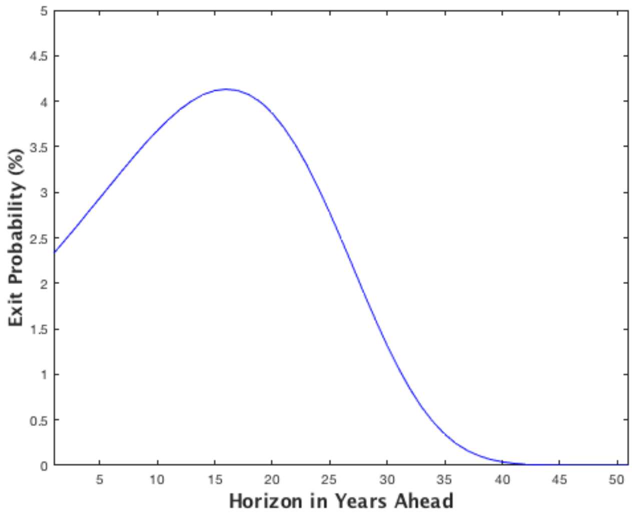
[4] It is often claimed that ERM portfolios (or exposure to ERM firms) are suitable for pension funds because they are long-term assets that are correlated with longevity risk. However, those who make such claims often overlook the exposure of ERM portfolios to housing risk. We will have more to say on this issue in a later report.

Figure 5 shows the exit probabilities for the baseline case. Having assumed away morbidity and early exercise, the exit probabilities equal the conditional mortality rates, i.e., the probability of death at future year t, conditional on surviving to that year. The exit probability for year t is equal to

(1) $$exit\ prob_t = q_t \times S_t$$

where $S_0 = 1$ and $S_t = q_{t-1}S_{t-1}$ for all $t > 0$. S_t is the probability that an individual alive now will survive to year t.

Figure 5: Base Case Exit Probabilities



Notes: Exit probabilities are based on M5-CBD model projections using male CMI male deaths rate data.

The early (low t) exit probabilities are close to the low t q_t rates and reflect the early high survival probabilities (i.e., that S_t is 1 or close to 1 for low t), and the later (high t) exit probabilities reflect the fact that the probabilities of living to extreme old age approach zero.

Valuation Issues

The present value ERM of the Equity Release loan can be considered to be the present value L of a risk-free loan, one which is guaranteed to be repaid in full, minus the present value $NNEG$ of the NNEG guarantee:

(2) $$ERM = L - NNEG.$$ [5]

The loan value grows at the loan rate l from its current amount until the time when the loan ends. Therefore L given by

(3) $$L = \sum_t [exit\ prob_t \times current\ loan\ amount \times e^{(l-r)t}]$$

[5] For more on NNEG and ERM valuation, see Li *et al.* (2010), Moreni and Mosconi (2013), Huang *et al.* (2016) and Gonçalves and Bravo (2018).

where $exit\ prob_t$ is the probability of exiting the house in period t, which we take to be the probability of death in period t, conditional on having survived to period t, and r is the risk-free rate.[6]

The valuation of L is straightforward.

$NNEG$ is the sum of the products of the exit probabilities for each future time t and the present value of the NNEG guarantee for each future time t:

(4) $$NNEG = \sum_t [exit\ prob_t \times NNEG_t]$$

where $NNEG_t$ is the present value of the NNEG guarantee for period t.

The question is then how to value each of these individual $NNEG_t$ terms and thence the NNEG guarantee.

Recall that the NNEG gives the customer (or the person acting for the customer) the right to repay the loan by paying the lender the minimum of the loan value or the house price at the time of death.

The right to repay the minimum of two future values (one of which, the future house price, is uncertain) at some given future time implies a European put option granted by the lender to the borrower. Since the time of exercise is uncertain, we can think of the NNEG as involving a portfolio of such put options.[7]

What approach should we use to value these options? One obvious possibility would be the vanilla Black-Scholes model (Black and Scholes, 1973; Merton, 1973). The problem with this model, however, is that it assumes that the underlying variable does not earn any yield. In the case of our put options the underlying variable is a house and it makes sense to think of a house as an asset that bears a continuous yield in the form of a rental rate. So if the house is worth £100k and has a rental rate of 3% a year, then the house generates a rental benefit of £3k a year. This rental benefit is the use-benefit of living in the house or the rental income we might get by renting the house out.

We need a model that allows for an underlying with a continuous rental benefit, and there are a number we could choose from – we could use the Black '76 model (Black, 1976), or we could tweak the Garman-Kohlhagen foreign currency option model (Garman and Kohlhagen, 1983) or we can use an appropriate special case of the Margrabe option, the

[6] Note the implicit distinction here between the loan amount or rolled up loan amount, on the one hand, and L, the (economic) value of the loan, on the other. The former is the amount loaned plus the interest accumulated since the inception of the loan, whereas the latter is the value of the loan to the lender, including the expected profit on the loan. A concrete example of the distinction between the two is given in Table 1. Note too that the economic value of the loan is not to be confused with the accounting book value of the loan, but we do not consider the accounting issues here.

[7] One might alternatively model the NNEG as a single American option with an early exercise feature but there would be no point in doing so. American options get interesting only when the option is exercised early in the self-interest of the option holder, but the decision to exercise early makes no sense in this context, because such a decision would be tantamount to the borrower taking their own life. It is then simplest to value the NNEG as an exit-prob-weighted average of the values of a set of European put options of differing maturities.

option to exchange one risky asset for another (Margrabe, 1978). These option pricing models are near-relatives of BS and are mathematically equivalent when applied to options on an asset with a continuous yield. We may as well then use the most straightforward model for our purpose and that is Black '76.

The Black '76 formula for the price p_t of a European put option with maturity t on a forward contract on a commodity bearing a continuous yield q is given by the formula:

(5) $$p_t = e^{-rt}[K_t N(-d_2) - F_t N(-d_1)]$$

where r is the risk-free rate of interest, K_t is the strike or exercise price for period t, F_t is the forward house price for period t, the function $N(\ldots)$ is the value of the cumulative standard normal distribution at the value specified in brackets, and d_1 and d_2 are given by:

(6) $$d_1 = [ln(F_t/K_t) + \sigma^2 t/2]/(\sigma\sqrt{t})$$

(7) $$d_2 = d_1 - \sigma\sqrt{t}$$

where σ is the volatility of the forward house price.[8]

The strike price K_t is then the rolled up or accumulated loan amount by period t:

(8) $$K_t = current\ loan\ amount\ \times e^{lt}$$

and the forward price F_t, the price agreed now to be paid on possession in period t, is:

(9) $$F_t = current\ house\ price\ \times e^{(r-q)t}$$

where q is the house net rental rate, i.e. the rental yield net of insurance costs, management costs, void and dilapidation.[9] This net rental rate is different from the 'headline' or gross rental yield, e.g., the amount received by a landlord. The net rental rate is also sometimes referred to in the property pricing literature as the deferment rate.[10]

F_t will decline as t gets longer, given that in current low interest rate conditions the risk-free rate r will be less than a plausible net rental yield.

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[8] Compared to the original Black-Scholes equation (Black and Scholes, 1973), we replace the spot underlying, the current house price, with the forward house price taken to be the current house price times $e^{(r-q)t}$. A point sometimes overlooked, the r term in the classic Black-Scholes formulas for d_1 and d_2 also drops out because the underlying contract is paid for at maturity and not at inception, and we assume that a rational seller would require compensation for growth on the sum of money they would have received if paid up front. This assumption should not be confused with the common misconception that the model assumes that the underlying grows at the risk-free rate.

[9] Note that this put option model is the same as that used by the PRA to value NNEGs (see PRA Consultation Paper CP 13/18, section 3.20).

[10] See Law Commission (England and Wales) CP 238 p. xii and section 14.47, which defines the deferment rate as "the annual discount applied, on a compound basis, to an anticipated future receipt (assessed at current prices) to arrive at its market value at an earlier date."

It is important to note that the forward house price F_t must not be confused with future house prices or expected future house prices:

- Forward prices for future period t are known (or can be approximated) now and we need to be able to value options using information available now.
- Options cannot be valued using future house prices because future house prices are currently unknown.
- Options should not be based on expected future prices because expectations of future prices do not appear in the Black '76 option pricing formula.

We should also keep in mind that although the original Black '76 article discussed options on futures, futures prices are the prices of *futures* contracts, a form of forward contract, *not* actual or expected *future* prices of any sort.

The mistake to be particularly avoided – the one common among UK ERM actuaries – is to confuse forward and expected future prices. This mistake typically manifests itself in the inputting of an assumed expected house price inflation rate into (8) instead of the forward rate $r - q$.

Finally, on the subject of the put pricing equation, it is helpful to use (9) to substitute out F_t from the price formula and replace it with the spot or current house price, S_0. Our put price formula is then:

(10) $$p_t = e^{-rt} K_t N(-d_2) - S_0\, e^{-qt} N(-d_1)$$

The term $S_0\, e^{-qt}$ is known as the *deferment house price*, the price we would agree and pay now for possession at a future time t. Equivalently, the deferment house price is the present value of the forward price, where the present value is obtained by discounting at the risk-free rate.

Note that the deferment house price will be less than the current house price S_0 because the net rental rate $q > 0$.

We would regard this point as obvious, but it turns out that not everyone sees it that way. The value of q turns out to be a central issue in the NNEG valuation controversy.

In recent discussions, we have sometimes encountered arguments that Black' 76 should not be used for one reason or another. We propose to address these arguments in a future paper. Suffice for the moment to note: (a) that such arguments are often based on a confusion of sufficient with necessary conditions for BS to hold; (b) that the 'holes in Black-Scholes' are well-known and options specialists know their way round them; (c) that to the extent that they might give different option valuations, plausible alternatives to Black '76 produce higher NNEG valuations than those produced by Black '76; and (d) that later in the paper we offer a model-free bounds analysis that enables us to place a lower bound on the NNEG valuation without relying on any option pricing model.

A Valuation Example

We now build an ERM and NNEG valuation model based on plausible input parameter values.

The baseline parameter inputs are:[11]

- Current age of customer = 70, a typical age for ERMs.[12]
- Loan to value ratio = 30%, which is sometimes suggested as typical for ERM loans to new customers aged 70.[13]
- Risk-free rate r = 1.5%.
- ERM loan rate l: recent ERM loans typically have interest rates in the range from 5% to 6%, although legacy rates can be higher. We assume 6%.
- Net rental rate q: Gross rental yields are usually about 5% of property value, and the net rental yield might be about 60% of total rental once one allows for management costs, void rate and dilapidation. We go with our "best estimate" of q = 3% although any q in the range from 2% to 4% seems reasonable (see also our Appendix A, IFoA, 2005 or Hosty *et alia* (2008, slide 22), which reports a q = 3.3%). We also agree with the PRA following CP 13/18 that any q less than 1% would be difficult to justify and that any $q \leq 0$ makes no sense.
- CP 13/18 section 2.16 states that PRA estimates of the volatility σ are in the range 10% to 15% and suggests a PRA "central estimate" of σ equal to 13%. This latter estimate seems reasonable and so we use it in our baseline calibrations.[14]

All rates are in % p.a.[15]

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[11] We gloss over some difficult issues relating to the absence of a reliable house price index (let alone, a reliable forward house price index) and the problems of basis risk and its relationship to plausible volatility assumptions. Suffice that it is reasonable to say that options specific to individual properties would justify higher rather than lower volatility assumptions. For more on these issues, see IFoA (2005) or Hosty *et alia* (2007).

[12] Implicitly, we are assuming a single male aged 70. In the case of a single female, we would expect death/exit to occur somewhat later, which would increase the value of the NNEG.

[13] A 30% LTV ratio for a 70-year old is often suggested as typical and is in line with an old 'age minus 40' rule of thumb for determining initial LTV based on age. Chart 5 in Rule (2018) suggests that actual LTVs are higher than that rule would suggest, and higher LTVs would lead to higher NNEG valuations than those we report here. The higher LTVs reported by Rule and due to the fact that LTVs increase over time as the ERM loan matures.

[14] An important caveat however is that this volatility is the volatility of the spot house price not that of the forward house price, and past experience suggests that the latter will exceed the former if we take account of interest rate volatility. The volatility of interest rates has been low since the Global Financial Crisis but was considerably higher before then. The interest rate volatility to be assumed going forward requires one to take a view about interest rate normalisation but suffice to note that we regard our volatility calibration as conservative, i.e., on the low side.

[15] We have not addressed fees or other expenses payable by the customer. Some idea of these can be seen from a specimen product specification given by Hosty *et al.* (2008, slide 9). These include distribution and market costs equal respectively to 2.5% and 1% of the advance, an initial charge of £500, an annual 'renewal' charge of £60 and a terminal charge of £350, and expense inflation of 3% p.a. Other costs (e.g., valuation and presumably disposal costs) are to be charged to the borrower. Updating these to 2018, making some illustrative assumptions (borrower age 70 with life expectancy of 16 further years, LTV = 30%, house price = £250k, house price inflation = 3.5% and applying a disposal cost of 2.5% of the terminal house price (a cost estimate suggested by IFoA (2005, p. 29), we estimated the present value of the charges and other costs applied to the borrower to be almost £13.k or 17.4% of the (nominal) advance of £75k. This estimate also ignores the costs to the borrower of the obligation to maintain the condition of the property to the lender's liking.

There are also other factors we have not addressed, but which might be addressed in a fuller analysis:

- The first is morbidity (or illness) risk in the last years of life. The customer might become too ill to remain in their home and might move into a nursing home before their death. This factor would bring forward the time of exit and likely produce a (somewhat) smaller NNEG valuation. See also footnote 17 below.
- The second is impaired lives: where customers are known to have medical conditions that affect their life expectancy, borrowers might offer them better terms (i.e., higher LTVs) than they would offer borrowers in normal health.
- The third is joint lives. In the case of a couple, the ERM contract would typically involve a joint lives policy, by which exit was deemed to occur when the remaining partner in the couple exited the house. A joint lives policy would delay expected exit and increase the value of the NNEG, and thereby tend to counterbalance morbidity
- The fourth is early redemption risk: some ERM contracts include early redemption clauses that allow customers to pay off their loans before exiting their house. In so far as these clauses grant customers a valuable option to pay off their loans early, they add to the lender's cost. On the other hand, early redemptions reduce the risk of negative equity which reduces $NNEG$. This latter effect will offset the first effect and the net effect is likely to be fairly small.[16]
- The fifth is term structure or, rather, term structures. We ignore the term structures of interest rates, q rates (but see Appendix A) and volatilities.

We assume an illustrative house price of £100 which, combined with the assumed loan to value ratio of 30%, implies a loan amount = £30.

The death/exit probabilities are derived from projections of future mortality rates obtained using the M5 version of the Cairns-Blake-Dowd mortality model (see Cairns *et alia*, 2006, 2009) calibrated on Continuous Mortality Improvement male mortality data for the period 1961 to 2015 and spanning ages 40 to 89. The M5-CBD model is particularly suitable for old age projections and its goodness of fit and performance evaluation are assessed in Cairns *et alia* (2011) and Dowd *et alia* (2010a,b).

Our baseline NNEG valuation results are shown in Table 1:

Table 1: Baseline ERM/NNEG Valuations

Current House Price	Loan Amount	L	$NNEG$	ERM
£100	£30	£65.3	£21.3	£44.0

Notes: L is the present value of the loan component of the Equity Release Mortgage, $NNEG$ is the is the present value of the NNEG guarantee, and ERM is the present value of the Equity Release Mortgage. Based on the baseline assumptions: male aged 70, LTV=30%, r=1.5%, l=6%, q=3%

[16] For conventional amortizing mortgages it is typical for authors to model prepayment explicitly as an option (e.g., Deng *et al*. 2000 or Chen *et al*. 2009). The analysis of prepayment for reverse mortgages/ERM is less well studied, but a recent analysis by Lee and Shi (2017) suggests that the value of the prepayment option is about 1.5% of the LTV. Some other illustrative numbers are reported in Hosty *et alia* (2008, slide 11). In correspondence, Matt Sekerke suggests an alternative approach in which the prepayment option can be modelled as the difference between a barrier option and a vanilla option such as Black '76.

and σ=13%. Exit probabilities are based on M5-CBD model projections using male CMI male deaths rate data spanning years 1961:2015 and ages 40:89.

Given the age of the customer, the expected present value L of the perfectly collateralised loan is £65.3. $NNEG$ is valued at £21.3 and so the value of the ERM, ERM, is equal to £65.33 – £21.3 = £44.0.

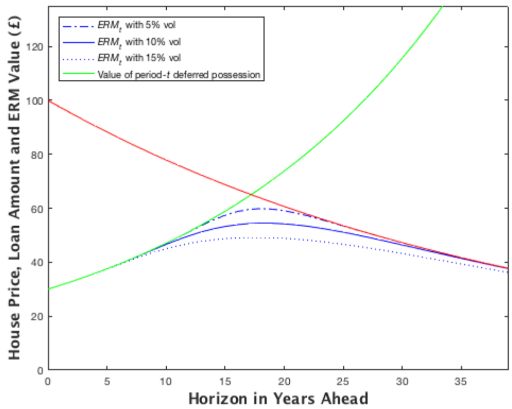
$NNEG$ is 21.3/30 = 71% of the loan amount or 21.3/44.0 = 48.4% of ERM.

An important caveat, however, is that the valuations reported in Table 1 can only be regarded as approximations because key parameters – in particular, r, q and σ – are not precisely known and have to be judgmentally calibrated.[17]

Sensitivities of Valuations to Key Input Parameters

It is interesting to examine further the impact of the volatility parameter. Figure 6 shows the impact of 3 different volatilities – $\sigma = 5\%$, $\sigma = 10\%$ and $\sigma = 15\%$ – on ERM_t. As volatility falls, the ERM_t curve moves up towards the upper bound L_t and deferment house price curves; the increase in ERM_t reflects the fall in the value of $NNEG_t$ as the volatility falls. Conversely, as volatility rises, ERM_t falls and $NNEG_t$ increases.

Figure 6: ERM and NNEG with Varying Volatility



Notes: As per Table 1 except for varying volatility.

Table 2 tabulates the values of L, $NNEG$ and ERM for the 3 different volatilities in the base case for age 70, current house price £100 and initial LTV = 30%. Again, the higher the volatility, the larger is $NNEG$ and the lower is ERM.

[17] As an aside, we can tweak this approach to produce a back of the envelope calculation that takes morbidity risk into account. Suppose that, on average, the borrower can be expected to go into a care home two years before they die. Suppose too that the life expectancy of a 70 year old male is, say, 17 years. He is therefore expected to live to 87 but to go into care at 85. If we want to use the mortality model to obtain the expected time to exit taking the expected time in care into account, then for modelling purposes we want to give the 70 year old two less years of life, giving him a life expectancy of 15 years. We can do that by treating him as if he were 72 years old. The baseline results for a 72 year old are then: L = £60.5 (down from £65.3), $NNEG$ = £16.6 (down from £21.3) and ERM = £44.0 (no change). This approach gives a rough estimate provided we have some idea of the expected time in care.

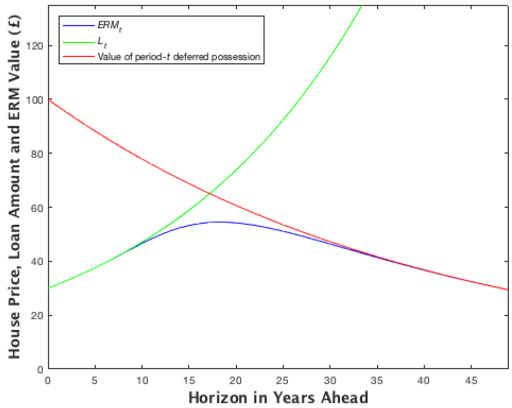
Table 2: L, $NNEG$ and ERM

Volatility σ	L	$NNEG$	ERM
$\sigma = 10\%$	£65.3	£19.7	£45.6
$\sigma = 13\%$	£65.3	£21.3	£44.0
$\sigma = 15\%$	£65.3	£22.5	£42.8

Notes: As per Table 1 except for specified changes.

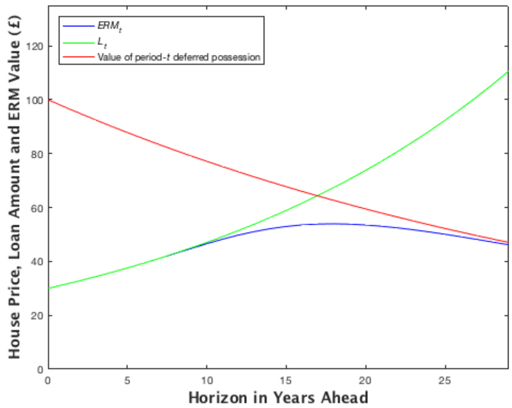
It is also interesting to examine further the impact of the borrower's age. Figures 7 and 8 give the L_t, period-t deferred possession value and ERM_t curves for ages 60 and 80. These figures correspond to the age 70 case dealt with in Figure 6.

Figure 7: ERM Value, Amount Loaned and Deferment House Prices: Age 60



Notes: As per Table 1 except for age 60.

Figure 8: ERM Value, Amount Loaned and Deferment House Prices: Age 80



Notes: As per Table 1 except for age 80.

Table 3 tabulates the results for ages 60, 70 and 80 for our base case volatility of 13%.

Table 3: Age, L, $NNEG$ and ERM

Age	L	$NNEG$	ERM
60	£100.5	£59.7	£40.8
70	£65.3	£21.3	£44.0
80	£46.7	£4.9	£41.8

Notes: As per Table 1 except for variations in ages.

The higher the age, the lower the future life expectancy and hence the lower is L. But $NNEG$ also falls with a higher age, and these two effects to a considerable extent offset each other in their net impact on ERM.[18]

Some Stress Test Results

We now consider some stress tests. A stress test is where one evaluates the impact of some hypothetical change on NNEG and ERM output values. Recall (2):

$$ERM = L - NNEG \quad (2)$$

In principle, a stress could involve changes to all three of these variables. The outcome of the stress test is then described by:

$$\Delta ERM = \Delta L - \Delta NNEG \quad (11)$$

where we use $'\Delta'$ to describe the change in the following variable.

Consider six potentially adverse stress test scenarios. Essentially, we assume that the ERM contract is entered into at base case assumptions, but then one of the following occurs:

- Stress test #1: The risk-free rate then falls to 0.5%.
- Stress test #2: Net rental rate rises from 3% to 4%.
- Stress test #3: Volatility rises from 13% to 15%.
- Stress test #4: House prices fall by 30%.
- Stress test #5: House prices fall by 40%.
- Stress test #6: Expected longevity increases by 2 years.

The results of these stress tests are shown in Table 4:

Table 4: Stress Test Results

Base estimate	$\boldsymbol{L}$	$\boldsymbol{NNEG}$	$\boldsymbol{ERM}$
	£65.3	£21.3	£44.0
Stress Test	**Change in $\boldsymbol{L}$**	**Change in $\boldsymbol{NNEG}$**	**Change in $\boldsymbol{ERM}$**
Stress test #1	£13.9	£11.3	£2.6
Stress test #2	£0	£4.1	-£4.1
Stress test #3	£0	£1.2	-£1.2
Stress test #4	£0	£7.4	-£7.4
Stress test #5	£0	£10.7	-£10.7
Stress test #6	£5.4	£5.6	-£0.2

Notes: Based on the baseline assumptions (see Notes to Table 1) with appropriate adjustments for each stress.

[18] Note that Figures 2 and 3 and Table 3 are meant to show the impact of changing borrower age other things equal, and are based on LTV = 30%. In practice, the LTV would rise with borrower age as explained in note 13.

The first line repeats the base case results, and the remaining lines show the impact of the assumed stress scenario on each of L, $NNEG$ and ERM.

Results are highly sensitive to the assumed stresses. Consequently, plausible stresses can *greatly increase* $NNEG$. In most cases considered, there is an equal and opposite impact on ERM.

An exception is the first case in which the risk-free interest rate falls from 1.5% to 0.5%. In this case, the loan value L also increases (because the discount factor falls) and the net impact is an increase in the ERM value. It is interesting to note that this net effect goes counter to the impression one might get from Figure 3 that an increase in longevity would inflict a loss on the lender. Figure 3 is correct that other things equal, the presence of the NNEG creates a loss to the lender, but the NNEG loss is mostly offset by an increase in L and this latter effect is not shown in Figure 3.

A second exception is the result of the longevity stress test #6.

The PRA's Good Practice ERM Valuation Principles

In its Supervisory Statement SS 3/17 published in July 2017, the UK Prudential Regulation Authority set out certain good practice principles relating to ERM portfolios. These principles include two that impose upper bounds on ERM valuations.

Principle II states:

> The economic value of ERM cash flows cannot be greater than either the value of an equivalent loan without an NNEG or the present value of deferred possession of the property providing collateral.

i.e.,

(12) $$ERM \leq L \text{ and } ERM \leq F$$

The first part of this statement follows from

(2) $$ERM = L - NNEG$$

and the fact that $NNEG \geq 0$. Hence

(13) $$ERM \leq L$$

and the first part is established.

The second part of this Principle II statement follows from the fact that the period t payoff from the forward contract is F_t, whereas the period t payoff to ERM_t is $\min[L_t, F_t]$. Therefore, one would never rationally pay more for the ERM_t payoff than one would pay for the forward.

Principle III states:

> The present value of deferred possession of a property should be less than the value of immediate possession

i.e.,

(14) Deferment house value < spot house value.

As the PRA explains, the rationale can be seen by comparing the value of two contracts, one giving immediate possession of the property, the other giving deferred possession when exit occurs. The only difference between these contracts is the value of foregone rights (e.g., to rental income or to use of the property) during the deferment period, and this value should be positive for the residential properties used as collateral for ERMs. It then follows that the present value of deferred possession should be less than the value of immediate possession, i.e., we obtain Principle III.

A more complete explanation of the validity of Principle III is given in Appendix B.

Misconceptions About the Good Practice Valuation Principles

This Principle III came under criticism from a surprising quarter. In June 2016, the Institute and Faculty of Actuaries issued "DP 1/16: Equity Release Mortgages: IFoA Response to the Prudential Regulation Authority," its official response to the PRA's earlier Discussion Paper DP 1/16, which had had asked for industry views on ERMs. To quote from this response:

> 33. For the second relationship in paragraph 4.9 [i.e., Principle III] to hold, in theory, there needs to be a deep and liquid market. Otherwise the implication is that the average value of the HPI [House Price Inflation] assumption is less than or equal to the discount rate assumed in the valuation of the NNEG. In practice, the approach to setting the HPI assumption varies significantly from firm to firm.

There are several mistakes here:

- Mistake #1 is that for Principle III "to hold, in theory, there needs to be a deep and liquid market." The validity of Principle III has nothing to do with a deep and liquid market and its validity holds under general conditions. But to spell it out a slightly different way. Consider two alternative contracts to be bought today. The first entitles us to immediate possession of a house, and therefore gives us rental services (i.e., the benefit from being able to live in the house or rent it out) from now till forever; the second entitles us to deferred possession of the same house at some future time t, and so gives us the same rental services from t onwards, but not the rental services from now to period t-1. Principle III states that one would pay less for the second product than for the first and follows from elementary economics. Why would we not pay less to get less?
- Mistake #2 is to suggest that the "average value of the HPI assumption is less than or equal to the discount rate assumed in the valuation of the NNEG." This statement is just plain wrong. The correct statement is that the (average or any)

value of the HPI assumption can be anything, but is always irrelevant to the valuation of the NNEG.

Para 35 then gives some illustrations of circumstances in which Principle III allegedly might not hold:

- One is the claim that Principle III "is a statement of 'value' and applies to any individual. However this is not necessarily true in terms of the exchange value." This strange statement is an imaginative addition to the economic theory of value but is unfortunately also wrong. The claim that the Principle III "is a statement of value and applies to any individual" is true, but the corollary is that it also applies to all individuals *including* (and not excluding!) when they engage in trade at market or exchange values.
- Another is the claim that "in a negative yield curve scenario, the relationship (Principle III) would fail as the premise that deferral could lead to a lower present value no longer holds." This statement is a real head scratcher but one can see that it must be wrong because the deferment price (or value, makes no difference here) is equal to $S_0 \, e^{-qt}$ and this expression does not include any interest rate or yield, negative or not. To repeat, Principle III depends only on the q rates being positive (or mostly positive) and it is difficult to imagine plausible situations where that would not be the case.

So how come the distinguished actuaries of the IFoA could make such mistakes? A possible clue is that the covering letter opens with the following statement:

> The IFoA's *Equity Release Members Interest Group* (ER MIG) and Life Board have been involved in the drafting of this response. The contributors to this response *include members who are actively engaged with use of equity release assets by life insurers*. (My italics)

The IFoA had allowed itself to be used as a mouthpiece for ERM industry leaders to broadcast their misunderstanding of their products in pursuit of their commercial interests.

But the authors of the IFoA official response to DP 1/16 are not alone in misunderstanding these principles. Consider these passages from a recent Deloitte communique on ERMs:

> In our view, the third principle (that future possession of a property cannot be more valuable than current possession) is likely to attract the most future debate.

But Principle III is just elementary economics!

> Very importantly, this principle implies that assumed future house price growth cannot exceed the discount rate applied in the valuation. …

No it does not.

> The PRA expects there to be a positive value associated with possession of a property.

Yes.

> The practical implication of this is that the assumed house price growth within the NNEG option pricing calculation cannot exceed the discount rate, as this would imply that future possession is more valuable.

No.

> This principle therefore effectively sets a cap on firms' house price growth assumptions.

No it does not! Principle III has *no* implications about assumed future house price growth. You can make any assumptions about future house price growth that you like and Principle III would be still be valid.

> We would expect firms investing in ERMs and other direct investments to see an increased level of scrutiny and questioning from the PRA, with the *bar set very high for management's understanding of the valuation of such investments*. (Bulley *et alia*, 2017, our italics)

They are clearly off to a flying start on that one.

The lead author, Andrew Bulley, is a partner in Deloitte's Centre for Regulatory Strategy. Prior to joining Deloitte, Mr. Bulley was Director of Insurance Supervision at the Bank of England.

To challenge Principle III is thus to make an egregious intellectual error and it is remarkable that the IFoA has not only failed to condemn any such challenge but has explicitly given it its imprimatur. This situation is analogous to the UK's top mathematical institute, the Institute of Mathematics and its Applications, taking the official view that the validity of 2+2=4 is an opinion. You see, some mathematicians are of the opinion that 2+2=4 but others have a different view.

We appear to have here another case of 'actuarial judgment' gone awry.

One is reminded of some comments made on this subject by Tim Gordon almost two decades ago (Gordon, 1999). He wrote (p. 4) about the actuarial conviction that "actuarial judgment is the only technique for valuing long-term liabilities" but 'actuarial judgement' produces an answer that "varies enormously depending on which actuary carries out the calculation." He continued:

> actuaries assume that judgmental methods are the only methods available which give sensible answers. What is more, the judgement involved is something which apparently only comes with years of experience. In other words, we claim to know the answer but cannot tell anyone else how to derive it in advance.

The experienced actuary knows it when they see it. Roman augurs had the same skill reading chicken entrails. As he continued further:

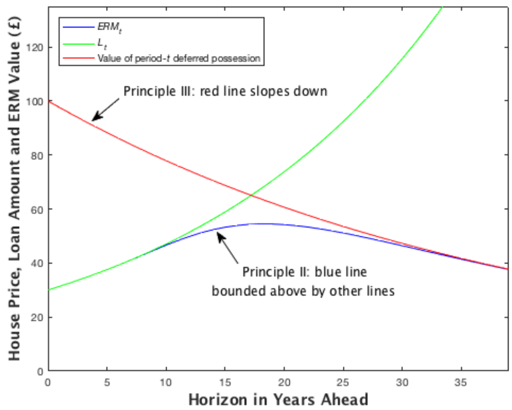
> The problem is that the difference that actuarial judgement can make to valuations using the traditional approach is enormous. It means that:
>
> • we are exposed to pressure from clients seeking to move answers in the direction which favours them, and
> • we lose credibility because we are unable to explain precisely how we arrive at an answer.

Actuarial judgment can also lose credibility when it produces answers that are demonstrably wrong.

Bounds on ERM and NNEG Valuations

To return to the main storyline, the impact of these two Principles is illustrated in Figure 9:

Figure 9: Illustration of Principles II and III



Notes: As per Table 1.

Principle II implies that the blue (ERM_t) line must be below both the green (L_t) line and the red (deferred possession) line, and Principle III implies that the red (deferred possession) line should slope downwards.

There is some interesting intuition underlying the Figure:

- For very low horizons, $NNEG_t$ is very out of the money and probability of exercise is very low. Hence the value of the option will be negligible and ERM_t will be indistinguishably close to the value of the loan L_t.
- For long horizons or high t, the option is well into the money and the probability of exercise is high and approaching 1. Therefore, the ERM_t line converges to the deferred house value line for period t.

Underlying these graphs are some elegant mathematics. ERM_t is given by

(15) $$p_t = e^{-rt}L_t - e^{-rt}[L_t N(-d_2) - F_t N(-d_1)]$$
$$= e^{-rt}[1 - N(-d_2)]L_t + N(-d_1)e^{-rt}F_t$$

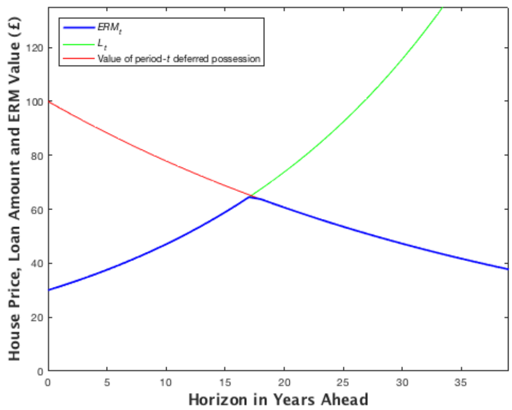
where we have set the deferment price $D_t = e^{-rt}F_t$. Now note the standard equivalence $N(-x) = 1 - N(x)$. We then get

(16) $$p_t = N(d_2)e^{-rt}L_t + N(-d_1)e^{-rt}F_t$$

This expression is simpler and reflects the shapes of the curves clearly. As d_2 gets positive, $-d_1$ gets negative, so $N(d_2)$ goes to $1, N(-d_1)$ goes to zero and ERM_t approaches the present value of the loan. As d_2 goes negative, it's the other way round, so the term on the left disappears and the term on the right approaches the deferment value $e^{-rt}F_t$. One sees these bounds at play in Figure 9.

Besides their mathematical elegance, these bounds implied by Principles II and III have a helpful practical use: they are an easily calculated cross-check on any proposed ERM or NNEG valuation. Consider Figure 10, which shows the upper bound for ERM_t made explicit and highlighted in blue.

Figure 10: ERM Upper Bound



Notes: As per Table 1.

We can obtain the ERM_t upper bound as the minimum of $e^{-rt}L_t$ and $e^{-rt}F_t$. Note that this upper bound can be estimated using only information about the current house price and LTV (which together give us the current amount loaned), the risk-free rate r, the net rental q, the loan rate l and the exit probabilities. For example, in the baseline case, we estimate the ERM upper bound to be £48.3, which compares to our earlier baseline estimate of ERM as £44.0. So even without estimating ERM or its *NNEG or estimating any underlying option model or calibrating any additional parameters* (e.g., as such as the volatility in an option-pricing model), we immediately know that any proposed value of ERM that exceeds £48.3 must be wrong.

But if we can estimate an upper bound for ERM without requiring an option-pricing model or relying on any volatility parameters, then by (2):

(2) $$ERM = L - NNEG$$

we can also estimate a lower bound for $NNEG$ on the same basis. Given that L = £65.3 in our baseline case, the upper bound ERM estimate of £48.3 implies a $NNEG$ lower bound equal to £17.0. This lower bound compares to our earlier $NNEG$ estimate of £21.3. So *even without estimating the NNEG or relying on any NNEG valuation model or any volatility estimate that might go into any such model*, we know that any proposed NNEG value below £17.0 must be wrong.

To cut to the chase, given these various inputs – the assumed age and gender, the assumed house price and LTV, the assumed r, q, and l rates, and the inputted exit probabilities – it is *impossible* to get a NNEG value any lower than £17.0 *whatever option pricing model one might use and regardless of how it might be calibrated*.

At the risk of repeating ourselves, we would stress that this lower bound NNEG value is *not* dependent on Black '76 and we recognise that the validity of Black '76 is controversial. The recent Institute reply to CP 13/18 released on 28 Sep 2018 made a great deal of noise about how autocorrelation, mean reversion, lack of Geometric Brownian Motion etc undermined the validity of Black '76, for example, and a number of participants at the LSE ~~punch-up~~ seminar on 1 October 2018 made similar points. We would dispute the validity of some of these claims – not least because they often confuse sufficient with necessary conditions for Black-Scholes type valuations to be valid – but even if these claims were all valid, they do not apply to the bounds-based valuation offered here, because that argument is not dependent on any option pricing at all, Black '76 or otherwise. For more on these issues, see Buckner (2018c).

We have here a handy cross-check of any proposed NNEG valuation.

Black' 76 vs. Principles-based bounds results

It is interesting to compare our baseline NNEG valuation results with the results we would have obtained had we dispensed with the option pricing model and used the Principles-based bounds instead:

Table 5: Baseline ERM/NNEG vs. Bounds Valuations

Current House Price	**Loan Amount**	L	$NNEG$	ERM
£100	£30	£65.3	£21.3	£44.0
Current House Price	**Loan Amount**	L	$NNEG$ **lower bound**	ERM **upper bound**
£100	£30	£65.3	£17.0	£48.3

Notes: As per Table 1.

The $NNEG$ lower bound is 17.0/30 = 56.7% of the loan amount or 17.0/48.3 = 35.2% of the ERM upper bound.

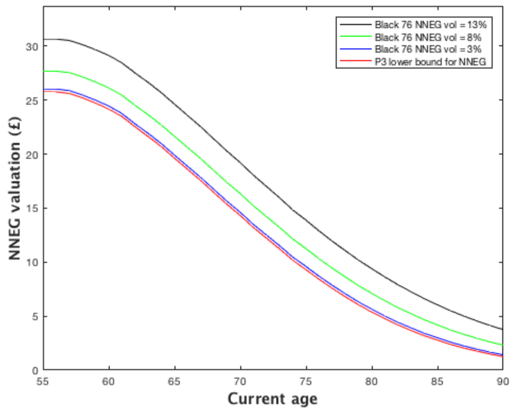
These results indicate that the basic NNEG under-valuation story obtained earlier using Black' 76 *still holds true if we use the Principles-based bounds instead of any option pricing model*.

Put another way, one cannot dismiss the NNEG under-valuation story based on arguments – right or wrong makes no difference – about the validity of Black '76 in this context.

Black' 76 valuation approaches NNEG lower bound as volatility gets small

Figure 11 plots various Black '76 NNEG valuations and the Principle 3 lower bounds for the range of ages from 55 to 90, where we assume that the LTVs are based on the borrowers' 'age minus 40' rule of thumb. We see that as the volatility gets small, the Black '76 NNEG valuations approach the lower bound from above.

Figure 11: The Black '76 NNEG and the NNEG Lower Bound



Notes: LTVs as per the 'age minus 40' rule of thumb. Volatilities as indicated. Otherwise as per Table 1.

If we wished to, we could approximate the NNEG lower bound by using Black' 76 with an extremely small volatility.

The $NNEG$ Lower Bound and the Forward Rate q

The $NNEG$ lower bound is acutely sensitive to the assumed q rate. Consider the results in Table 6:

Table 6: The Net Rental Rate q and the $NNEG$ Lower Bound

Assumed q	Comment on Assumed q	$NNEG$ lower Bound	$NNEG$ Lower Bound /Amount Loaned
$q = 0\%$	Impossible because $q \leq 0$ violates Principle III	£1.9	6.4%
$q = 1\%$	Low estimate	£6.1	20.4%
$q = 2\%$	Reasonable estimate?	£11.5	38.4%
$q = 3\%$	Best estimate?	£17.0	56.8%
$q = 4\%$	Reasonable estimate?	£22.2	74.1%

Notes: As per Table 1 except for changes in q.

A reasonable estimate of q might be 2% to 4%. A q in this range would give us a NNEG that is somewhere between 38.4% and 74.1% of the amount loaned.

The assumed q rate is doing a lot of the heavy lifting.

Misvaluation Issues in the UK Equity Release Market

Turning now to the UK ERM sector, consider this passage from the Deloitte communique on ERMs that we quoted earlier:

> For an asset class that represents just 1.4% of insurers' asset holdings, equity release mortgages (ERMs) have consumed a remarkable amount of firm and supervisory time. A decade or so ago, the regulatory challenge of this asset class lay on the conduct side. More recently, however, and not without some irony, the main mitigant of these conduct risks, the no negative equity guarantee (NNEG), has switched the focus primarily onto the inherent prudential risks of equity release, namely its illiquidity and, owing to the NNEG, the long term exposure it brings to the fortunes of the housing market without further recourse to the borrower. (Bulley *et al.*, 2017)

Similar concerns were expressed by David Rule, the PRA's executive director for insurance supervision. In a speech to the ABI in July 2017, Mr. Rule warned that the results of a PRA stress test had indicated that a 30% house price fall could lead to losses of £2 billion to £3 billion, or 8% to 12% of the ERM sector's assets, with the exposures skewed towards firms with larger house price or ERM exposure. More recently (April 2018, p. 5), Mr. Rule went further, saying that ERM books "could face difficulties in scenarios of flat, as well as falling, nominal house prices."

These statements come in a context in which the PRA has issued a number of letters going back to October 2014 and resulting consultation papers, discussion papers and supervisory statements (see, e.g., DP 1/16, CP 48/16, CP 23/17, CP 24/17, SS 3/17 and CP 13/18) that had set out a number of concerns about ERM firms' exposures and modelling practices. The number, scale and intensity of these documents suggest that regulators have been worried about ERM modelling practices since at least 2014. As CP 48/16 drily noted (pp. 6, 19), there is

> a wide variety of practice regarding valuation of the embedded guarantee, with suggestions that sometimes diverged from conventional approaches to the valuation of guarantees in incomplete markets." […]
>
> [But there] was consensus that property assumptions (growth and volatility) were most significant [in the valuation of the NNEG].

In plain English, firms were all over the place on NNEG valuation, which is a source of concern in itself, but there was a consensus on the relevance of property growth assumptions. This consensus is an even bigger concern, because (expected) property price growth is *irrelevant* to option pricing. Recall that the property growth or expected property growth does not appear in any Black-Scholes-type pricing equations.

Here are some samples of five firms' statements about their NNEG valuation approaches from their recent (2016) reports:

> "When calculating the value of the no-negative equity guarantee on the lifetime mortgages, certain economic assumptions are required within the variant of the Black-Scholes formula. […] In the absence of a reliable long-term forward curve for UK residential property price inflation, the [firm] has made an assumption about future residential property price inflation. … This results in a single rate of future house price growth of 4.25%."[19]

> "[The value of the NNEG] is calculated using a variant of the Black Scholes option pricing model. The key assumptions used to derive the value of the no-negative equity guarantee include current property price, *property growth* and property volatility."

> "Stochastic modelling is used to capture the expected cost of [the NNEG], which will depend on the expected rate and volatility of *future house price growth* …

> "Equity release and securitised mortgage loans … are valued using an internal model. Inputs to the model include primarily *property growth rates*, mortality and morbidity assumptions, …."

> "The fair value of the guarantee is determined using a stochastic model. The fair value of the loans is determined using assumptions for interest rates, *future house price inflation* and its volatility …"

So instead of using some proxy for forward house prices, which would have been the sensible approach, these firms used a projection of future house price growth. Their use of an irrelevant variable indicates that they cannot be valuing their NNEGs properly.

To their credit, the PRA picked up on this problem. Referring to the results of an earlier survey, CP 48/16 states (p. 25):

> Many respondents mentioned a version of the Black-Scholes formula known as 'Black 76', where the underlying price is the 'forward price' of the property. This version uses the current price of a forward contract. Some respondents appeared to *conflate* this with the forecast future price of the property, but provided *no justification* for why house price inflation was relevant to the current price of a forward contract. (My italics)

The key word is "conflate". The reason why these correspondents provided no justification for using projections of future house price inflation to value these guarantees is because no such justification exists.

[19] This calibration and significance of this parameter have attracted considerable attention. We address these issues further in Appendix C.

To spell it out: some firms say that they are using *assumptions about future house price growth*, but the PRA correctly says that this is *obviously wrong*. From which it follows (1) that some firms are using a method wholly at odds with the one endorsed by the PRA and (2) that the PRA would not be bothering to state this at all, particularly through a protracted consultation period if it had not experienced substantial pushback from firms. We can then infer (3) that firms with equity release exposure have been undervaluing their no negative equity guarantees. We can infer this because the PRA would not be publishing on the subject or seeking industry consultation if they thought that these guarantees were correctly valued. Consequently, some firms are presumably undervaluing them. Also (4) by a similar logic, if firms are dedicating substantial resources to pushing back, they must think that the valuation of guarantees is a material issue.

In fact, we are not aware of a *single firm* that has *demonstrated* that it is valuing its NNEGs using a defensible methodology.

The IFoA on NNEG Valuation

So the industry is riddled with misconceptions about NNEG valuation, but where do these misconceptions come from?

The IFoA 2005 report

In 2005, the IFoA published a report on NNEG valuation (IFoA, 2005). The gist of the report was that NNEGs should be valued using a Black-Scholes methodology along the lines used in this article. Roughly speaking, we can interpret the report as suggesting that the BS methodology provides a reasonable lower bound estimate of the value of the NNEG. We agree.

The report then offers some example valuations based on a then-plausible set of input parameters. To be precise, they assume: risk-free return = 4.75%, net rental yield = 2%, l=7.5%, disposal costs at sale = 2.5%, volatility = 12% and LTVs equal to 17%, 27% and 37% for ages 60, 70 and 80 respectively. Note that their approach, like ours, uses forward house prices obtained as the spot house price compounded at the forward rate $r - q$, although they do not use the term "forward house price".

Table 7 gives their NNEG valuations for hypothetical case of 60, 70 and 80 year-old males with ERMs:

Table 7: NNEG Valuations as % of Cash Advanced

Male aged	IFoA (2005) Table 3.13	Our model calibrated to their parameters
60	31%	31%
70	20%	16.9%
80	8.7%	7.0%

Notes: Right-hand side results based on: LTV=17%, 27% and 37% for ages 60, 70 and 80 respectively, r=4.75%, l=7.5%, q=2%, σ=12% and disposal costs = 2%. Exit probabilities are based on M5-CBD model projections using male CMI male deaths rate data.

The two models – theirs and ours – produce results that are fairly close, and the small differences between them can presumably be mostly ascribed to differences in their approaches to the modelling of the mortality/exit probabilities.

The IFoA report explicitly refrained from endorsing any particular "best-estimate" of future house price inflation:

> We assume that providers of property reversions either have an appetite for house price inflation exposure (i.e. they want to invest in properties), or that they will place the exposure with someone who has such an appetite. *This therefore becomes an investment decision* pertinent to the provider's own circumstances, *rather than a pricing risk*. Accordingly, we have not analysed the exposure in detail here, and in particular *we have resisted any temptation to suggest a "best-estimate" property growth rate!*" (IFoA, 2005, p. 32, my italics)

It might have been more helpful had the IFoA report suggested that it had resisted the temptation to produce a "best estimate" of future property growth on the grounds that no such estimate would have been of any use in option valuation.

It then noted (p. 32) that this "market consistent" approach "is not without its difficulties and shortcomings. We believe however, that our approach is consistent with the approach that many life offices are currently adopting in establishing their market-consistent or realistic liabilities."

Ominously, in light of later developments, it also noted that, "Others may however, prefer to approach the assessment of the NNEG using more of a "real world" stochastic modelling approach," whatever that might be.

And so we have the juxtaposition of BS as a reasonable approach to "market consistent" NNEG valuation, and an alternative unspecified "real world" approach that is pulled out of thin air and gives a different valuation.

In this latter approach, we have here the seed of a toxic weed that would soon germinate.

Hosty et alia (2007)

Two years later, the IFoA issued another report on NNEG valuation, Hosty *et alia* (2007). This report started with some concerns about the decline of profitability and its impact on the development of the ERM market

> the competitive environment that has driven product innovation has … resulted in lower product margins. This is all good for the consumer, but it is increasingly difficult for providers to reach target returns on capital, and this is deterring some prospective new entrants. One of the purposes of this paper is to investigate the profitability of typical schemes in the market at present, and so to address the question of whether competition has forced the market to function at non-profitable levels. … We will aim to provide a rational pricing methodology which can be adopted by any organisation active in the market,

> and we hope that this can support the market as it expands over the coming years.
>
> There is now concern that providers *may not be able to offer a product profitably at current margins*. Some competitive pressure is clearly a good thing, as it will force providers to find more efficient ways of providing their product to consumers. In the equity release market, too much competitive pressure may be a bad thing. (pp. 1-2, our emphasis)

To put it this way: their main concern is that overly high NNEG valuations might undermine the ability of firms to meet their profit targets.

They then examine two alternative approaches to NNEG valuation. The first (in their section 7.3.1) examines their version of a "market consistent" approach. The term "market consistent" is not defined and the nearest we get to an explanation is that this approach is based on an

> approximate market consistent basis similar to the pricing of options on stocks. … The main challenge with a market consistent basis is the fact that there is no underlying market to speak of. Accordingly we have tried to create a proxy market consistent basis using techniques that are standard in similar markets, specifically Black Scholes style modelling. (p. 26)

It further explains this approach a little later:

> Using a risk neutral basis, *house price inflation* should be linked to the return on long term risk free instruments (i.e. government stocks) less an assumption for rental income (net of expenses). (p. 26, our emphasis)

Now this verbal explanation might have been fine if they had just used the term "forward house prices" instead of "house price inflation". The use of the latter term is unfortunate, as it suggests that the house price inflation rate is the underlying variable in the BS model, and we have already seen that the house price inflation rate is irrelevant. Instead, BS or, more precisely, Black '76 tells us that the underlying variable that *should* go into the option pricing equation is the forward price, in this case, the forward house price. The use of term "house price inflation" in this context suggests a serious misunderstanding of how BS option pricing works.

Despite this misunderstanding, it would appear that the authors applied a correct formula (i.e., they took the current house price and compounded it at the forward rate $r - q$) to produce a forward house price series that they mistook for (or mislabeled as) a house price inflation series.

So their "market consistent" approach is essentially correct except for this (important!) misunderstanding/mislabeling.

But Hosty *et alia* make it plain that they do not like this "market consistent" approach:

> In reality the absence of an underlying market means that this proxy market consistent approach is only of limited academic value … (p. 27, our italics)

By "absence of an underlying market" they mean the absence of a liquid market in which the option can be hedged using a zero-arbitrage trading strategy. This point raises important pricing methodology questions that we will come to in a later article, and let it suffice for the moment to note that this argument confuses sufficient with necessary conditions for BS to be valid. The "only of limited academic value" jibe is presumably meant to suggest that the MC approach – or "proxy market consistent approach" as they put it – is of no practical 'real world' use. We disagree: what else would one use?

Then they make a further criticism of the MC approach:

> For providers attempting to price the NNEG on a market consistent basis there is insufficient product margin in order to provide a competitive product unless they have strong competitive advantages in one or more of the other cost areas. (p. 30)

Whether or not this claim is true – we will come back to this issue below – this statement begs the central issue, i.e., whether the MC-based valuations are accurate or not, and Hosty *et al.* have provided no convincing grounds to regard MC-based valuations as inaccurate.

There is also another interesting issue here, i.e., does correct guarantee pricing make a product uncompetitive if competitors are under-valuing their guarantees? Our answer: quite possibly, in the short term. Those competitors – even one large competitor – could set up a race to the bottom, and firms that entered this race would be storing up problems for themselves and may have difficulty staying solvent long-term, especially if they have been making distributions based on over-estimated profits.

Be these issues as they may, Hosty *et ali*a's main objection to MC valuation boils down to it giving valuations that they don't like and never mind whether those valuations are accurate.

This is Equitable Life all over again, i.e., the undervaluation of long-term guarantees!

Section 7.3.2 examines their preferred alternative, an "insurance pricing basis using "real world'' assumptions." What these assumptions might be they do not explain; nor, do they explain what their "real world" approach even is.

Section 7.3.2 consists of only 143 words and is here reproduced in full:

> 7.3.2 *"Real world: assumptions"*
> The alternative method we have used is to calculate the option cost using "real world" basis. The methodology we have used is as follows:
> – Use the log normal model as before (with same volatility).
> – A best estimate of 4.5% p.a. for HPI in the future (see Section 4.4). This is then the mean return under the model.
> – We have assumed that a real world discount rate of 4.75% per annum.

- We have not assumed a “mean reversion'' so that the random walk in each future period is applied independently of the position is [sic] preceding periods. The authors acknowledge that use of a “mean reversion” approach is equally valid.
- Results for sample model points and our overall portfolio are in Table 7.3.2. [KD: I have not reproduced this in full, but see Tables 8 and 9 below.]

> As can be seen, the resulting costs are significantly below those assessed using our proxy market consistent basis.

So not a single word of explanation as to why we should regard this RW approach as accurate, but the phrase that jumps out is “A best estimate of 4.5% for HPI in the future,” i.e., the RW approach is based on a guess about future HPI!

Based on the limited information provided, their “real world” approach would appear to be similar to the MC approach, but with the forward house price replaced by some assumed expected future HPI.

We now see the seed germinate. The 2005 IFoA report introduced the Trojan Horse of “house price inflation,” but at least did the calculations correctly. This error could be forgiven as an innocuous terminological one, except that the passage quoted opens the door to full-scale misuse and seems to confirm that the Hosty *et alia* 2007 “real world” valuation approach is based on that error, lock stock and barrel. The inclusion of HPI is no longer a mere mislabeling, but a bedrock principle of the RW approach.

To spell it out, HPI is now a key input *in its own right*.

Which points confirm that this approach is inconsistent with option pricing theory and therefore wrong.

Section 7.3.3 clarifies the authors’ views on which approach is to be preferred. I reproduce part of it here with some comments:

> *7.3.3 Market consistent or real world?*
> On our proxy market consistent approach we have derived a cost for the NNEG which would render the product non-profitable, whilst real world modelling has produced a significantly lower cost.

Or to quote a succinct bullet point from their 2008 presentation:

> If NNEG on a market consistent basis, unlikely to be profitable (Hosty *et alia*, 2008, slide 29)

There is no question that their “real world” approach produces a lower NNEG cost, but look where under-valuing long-term guarantees got Equitable!

To repeat: the issue is not to get a *low cost*, but an *accurate one*. Otherwise problems ensue. You can’t sustain a strong long-term business by selling guarantees that are under-valued.

To continue:

> Which [MC or RW] approach is most appropriate will depend on the purpose for which the analysis is being carried out. *For a realistic assessment of the cost of future negative equity claims, the real world approach is clearly the best approach* (My italics, p. 29)

So never mind the quality, feel the width.

But what about the NNEG valuations themselves?

Let's focus on our baseline 70 year-old male.

The results for a 70-year-old male are shown in Table 8:

Table 8: Hosty *et alia* "Market Consistent" NNEG Valuations as % of Cash Advanced: Male 70

Hosty *et alia* "market consistent"	Our reconstruction
12%	18%

Notes: 1. Source of left-hand side figure is Hosty *et alia* (2007, Table 7.3.1(a) for males aged 70. 2. They use the term "initial mortgage" rather than "cash advanced." 3. Key assumptions are: r = 4.75%, q = 3.3%, σ = 11%, LTV = 30%, l = 6.7% and house price disposal cost = 2%. 4. My reconstruction is based on M5-CBD model projections using male CMI male deaths rate data.

Our reconstructed number (18%) is somewhat higher than their 12%, but some of this difference is likely due to my model having lower mortality projections.

Table 9 reports the Hosty *et alia* "real world" NNEG valuation as a % of cash advanced, alongside our reconstruction based on the same input parameters.

Table 9: Hosty *et alia* "Real World" NNEG Valuations as % of Cash Advanced: Male Aged 70

Hosty *et alia* "real world"	Our reconstruction
1.8%	3.2%

Notes: 1. Source of left-hand side figure: Hosty *et alia* (2007, Table 7.3.2 for males aged 70. 2. They use the term "initial mortgage" rather than "cash advanced." Key assumptions are: r = 4.75%, q = 0.25%, σ = 11%, LTV = 30%, l = 6.7% and house price disposal cost = 2%. Exit probabilities are based on M5-CBD model projections using male CMI male deaths rate data.

One can see why they like their "real world" approach: their "real world" valuation is only 15% of their "market consistent" valuation!

Since we know that the BS valuations are respectable, we must conclude that the "real world" valuations are not.

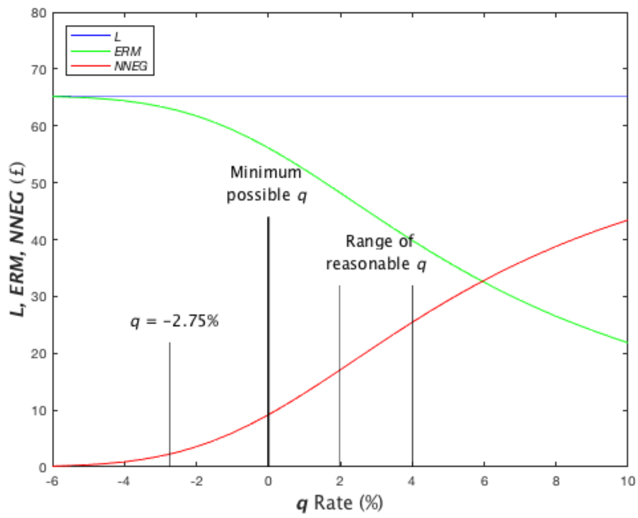
This conclusion also makes sense from a first-principles perspective. Let's forget the mis-references in Hosty *et alia* to the future house price inflation and so forth. The driving difference between their MC and RW calibrations is that the former has q = 3.3% whereas the latter has an implied q = 0.25%. The former falls within a reasonable q range of 2% to perhaps 4%, but the latter is well below this range.

It is exactly this excessively low q rate that drives the low NNEG valuations that Hosty *et alia* were promoting.

Rental Rates Revisited

To illustrate the point, Figure 12 shows a plot of L, ERM and $NNEG$ against q.

Figure 12: L, ERM and $NNEG$ vs q



Notes: As per Table 1, except for the range of q rates.

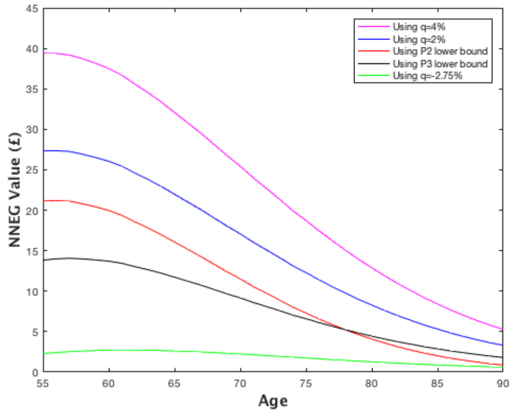
The Figure shows an illustrative q rate of $q = -2.75\%$ based on an expected HPI rate of 4.25%, a figure recently used by one of the big ERM firms, Just Group.[20] This assumed q rate leads to a very low NNEG valuation. The figure also shows the minimum possible q consistent with Principle III, i.e., a q epsilon above zero, and it is interesting to note that the q based on an expected 4.25% HPI rate and the minimum possible q are at odds.

And so it happens that the q rate is the critical factor in NNEG valuation and ones based on HPI rates are not so much implausibly but *impossibly* low.

To investigate further, Figure 13 shows plots of NNEG valuation against age under alternative assumptions about q, under the assumption that the LTV ratios are based on the 'age minus 40' rule of thumb, i.e., the LTV ratios start at 15% for age 55 and go up to 50% for age 90.

[20] Assume that the forward rate $f = r - q = 4.25\%$. Given our assumption of $r = 1.5\%$, then we obtain an implied $q = r - f = -2.75\%$.

Figure 13: $NNEG$ vs. Age Under Alternative q Assumptions

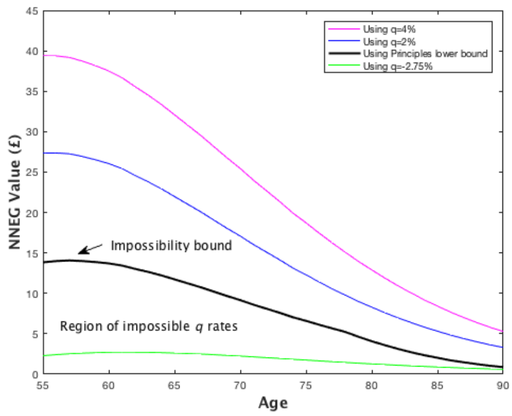


Notes: As per Figure 12 and assuming the 'age minus 40' rule of thumb for LTV ratios.

The top curve plots Black' 76-based NNEG valuations under the assumption that q = 4%, which we take the top of the range of reasonable q rates. The blue curve plots comparable NNEG valuations under the assumption that q = 2%, which we take to be the bottom of the range of reasonable q rates. The red curve plots lower bound NNEG valuations under Principle 2. The black curve plots NNEG valuations under the assumption that q is the absolute minimum allowed by Principle 3, which is epsilon above 0%. Finally, the green curve along the bottom of the Figure plots the (Black' 76) NNEG valuations obtained from our illustrative HPI-based q rate, i.e., $q = -2.75\%$.

To help the interpretation of these results, Figure 14 shows a slightly simplified version of Figure 14 in which the minimum NEG valuations permitted under Principles 2 and 3 are collapsed into a single 'Principles' lower bound, which is the minimum of the Principle 2 lower bound and the Principle 3 lower bound. This lower bound plot is shown in thick black, and by definition, any NNEG valuations below this line are impossible given the other calibrations.

Figure 14: Reasonable, Minimum Possible and HPI-Based NNEG Valuations

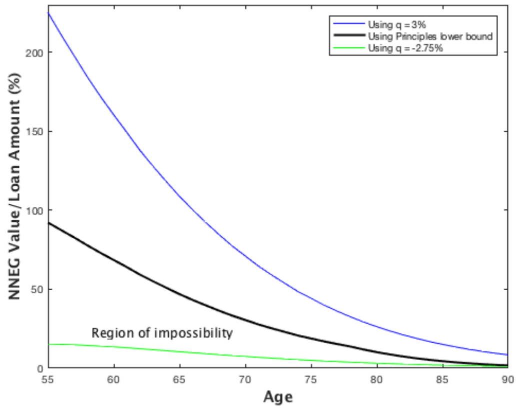


Notes: As per Figure 13.

We see that the green HPI-based NNEG valuation curve lies well below the minimum possible bound, the 'impossibility bound'.

Finally, Figure 15 shows a simplified version of Figure 14 but with the NNEG value on the vertical axis replaced by NNEG value divided by the loan amount:

Figure 15: NNEG/Loan Amount vs. Age



Notes: As per Figure 13.

Lessons Learned from the Equitable Case

In the aftermath of the Equitable Life case, there was much soul-searching in the UK actuarial profession and the Government commissioned Sir Derek Morris, the former head of the Competition Commission, to look into the failings of the profession and make recommendations for reform. The message from his report was unequivocal and hard-hitting: there needed to be greater scrutiny of actuaries' performance, and broader education and training:

> 4.4 In its interim assessment the review highlighted concerns about the process by which the Profession has sought to keep its syllabus, and associated teaching materials, up to date. Thus, for example, the perceived failure to adopt latest developments in financial economics and financial markets was seen in large part to stem from the role played by entrenched commercial interests in the development of the Profession's education policy and an insularity that constrained the extent and effectiveness of input from academics, other professions and those in wider fields of practice. (Morris, 2005)

> The UK now finds itself in a position where there has been a flight from, and resistance to, a range of financial products which arguably consumers should purchase to maximise their lifetime economic well-being. It is not unreasonable to argue that of all those involved, actuaries were the most appropriately trained experts who should have provided the expertise necessary to avoid this situation. … They were not sufficiently innovative ... they remained too locked in the environment of the 1970s and 1980s and too persuaded of their own abilities .... (Quoted in Thornton, 2004)

The response of the Institute of Actuaries was summed up by its president, Michael Pomeroy: "A lot of the events described in this report took place in the late 1980s and we are now almost in 2005 so we are a different profession."[21] Problem fixed.

By a curious coincidence, the IFoA has just (6 August 2018, i.e., the day before the BBC and Adam Smith Institute reports came out) issued its submission to the Kingman Review into the Financial Reporting Council (FRC). The Kingman Review is an independent review recently ordered by the Government after a number of high profile corporate collapses (most notably that of Carillion) which called into question the effectiveness of the FRC. Its chair, Sir John Kingman, is a former Second Permanent Secretary to the Treasury and is currently the Chairman of Legal & General, which, by another coincidence, is also a leading player in the ERM sector.

Here are the salient points from the IFoA submission:

> 3. The actuarial profession in the UK and the IFoA have developed significantly since the Morris Review in 2005.
>
> 4. In particular, the IFoA has introduced significant changes to create a robust and transparent regulatory framework. …
>
> 13. The IFoA is, we believe, seen as a leader in terms of actuarial regulation and qualifications by other actuarial associations around the world and has one of the most sophisticated regulatory and qualification frameworks of any actuarial professional body.
>
> 17. In general, we believe that the regulatory, qualification and oversight arrangements in relation to actuaries put in place following the Morris Review remain appropriate.
>
> 28. The IFoA believes that the model of professional self-regulation subject to effective independent oversight remains the most appropriate arrangement for the regulation of actuaries in the UK.
>
> 29. It means that the professional body for actuaries in the UK can provide insight and knowledge, as well as resource, to ensure that the regulatory and qualification framework for actuaries is relevant and effective while ensuring, through independent oversight, that this activity is carried out, and can be seen to be carried out, in a way that serves the public interest.
>
> 62. There is no evidence to suggest that the current arrangements are not serving to protect the public interest …

All very reassuring, especially the absence of any concerns about dodgy put valuation models or of the most significant weakness found by Morris, namely "professional

[21] Quoted in Thornton, 2004.

standards that have been weak, ambiguous or too limited in range, and perceived as influenced by commercial interests."

As the IFoA's website explains:

> The Institute and Faculty of Actuaries (IFoA) responds to consultations on a wide range of issues to support its members and to fulfil our *public interest role* ...
>
> The IFoA's focus is on responding to those consultations where actuarial expertise can inform debate, especially when the proposals will also directly impact on actuaries' working lives and/or *there are strong public interest issues involved*. (Our emphasis)[22]

The IFoA is not some shabby industry lobbying group. The IFoA is a body representing a profession, with the professional standards that go with it. It also has a Royal Charter that imposes a duty on it to "put the public interest first."

This duty includes an obligation to maintain professional standards. To quote the IFoA website: "Actuaries must comply with *rigorous* professional standards"[23] and one would presume that "rigorous professional standards" would include an obligation to use a scientifically respectable approach to NNEG valuation and to refrain from using any approach that produces, e.g., impossible results.

We address these technical standards issues further in Appendix D.

The duty to promote the public interest also imposes on actuaries the obligation to be impartial and avoid *even the appearance* of conflicts of interest:

> Conflicts of interest is one of the key regulatory areas for the Institute and Faculty of Actuaries (IFoA) and its members ...
>
> As one of the five key principles of the Actuaries' Code [of Conduct], impartiality is placed in sharp focus in the context of professional conflicts of interest, *actual or perceived* ...
>
> Impartiality in the context of the Actuaries' Code means that: members will not allow bias, conflicts of interest, or the undue influence of others to override their professional judgement.[24] (Our emphasis)

All very admirable, but compare these statements to the IFoA's response to DP 1/16 which we quoted earlier:

> The IFoA's *Equity Release Members Interest Group* (ER MIG) and Life Board have been involved in the drafting of this response. The contributors to this

[22] https://www.actuaries.org.uk/news-and-insights/consultation-responses.
[23] See https://www.actuaries.org.uk/upholding-standards, our emphasis).
[24] https://www.actuaries.org.uk/upholding-standards/conflicts-interest.

> response *include members who are actively engaged with use of equity release assets by life insurers*. (Our emphasis)

Isn't there at least the possibility of a perception of possible conflict of interest here?

Conclusions

It is often said that the Equitable Life scandal taught us about the dangers of under-valuing opaque long-term guarantees. The under-valuation of these guarantees was good for business in the short-term, but lethal for Equitable in the long-term.

In the aftermath we were assured that lessons had been learned etc. and the vast hugely expensive Solvency II apparatus was introduced to prevent a similar fiasco in the future.

Twenty years on, the same problem has re-emerged in the ERM sector: not yet two years into operation and Solvency II has already failed.

In this case the under-valued guarantees are the NNEGs in the Equity Release contracts. The extent of these under-valuations is hardly trivial – our results suggest that NNEGs might be under-estimated by *an order of magnitude* in some cases and we are not aware of a single firm in the UK ERM sector that has demonstrated that it is valuing its NNEGs using a respectable valuation methodology. As with Equitable Life, under-valued guarantees imply overstated profits and raise questions about the financial conditions of the firms involved.

Despite the facts that NNEGs are a form of option and that the principles of sound option pricing have been known for forty-five years, practitioners in the UK ERM sector are still wedded to a bogus approach that has no scientific justification and has not had a single endorsement from a recognised independent expert.

In the equity release sector at least, there is a continued failure to come to terms with the insights of modern finance theory. A significant section of the actuarial profession resemble those who persisted with astrolabic studies in the face of all the scientific explanation coming from the likes of Copernicus, Galileo, etc, because they did not like the results that science was giving them.

There is also evidence that the Institute is still being used as a vehicle for commercial interests, even though such abuse was one of the key concerns of the Morris Review.

There are clearly lessons to be learned and these are the same lessons that weren't learned before.

Appendix A: The Calibration of the Net Rental Rate

In the text we stated that "We go with our "best estimate" of $q = 3\%$ although any q in the range from 2% to 4% seems reasonable."

This Appendix provides evidence to support a 'best estimate' q rate that is of the order of at least 3%.

Let's consider a 'micro example' first. This evidence comes from estimates based on data from the transcript of the Upper Tribunal (Lands Chamber), January 2018 ('Trustees of the Sloane Stanley Estate versus Adrian Howard Mundy [et al]'). Lessor and lessees submitted claims for *leasehold relativity*, namely their estimate of current leasehold value divided by 'freehold vacant possession value' (FHVP), which is an estimate of the value of the property unencumbered by the freehold interest. We took these estimates, and on the assumption that the sum of freehold and leasehold relativity – the relative shares of freehold and leasehold in the value of the property – is equal to 100%, i.e. assuming that there is *no* marriage value (the additional value an interest in land gains when the landlord's and the leaseholder's separate interests are "married" into single ownership, see Law Commission, 2018, p. 23, n. 62). This approach gives a rough estimate of the q rates or deferment rates claimed by lessee and lessor.[25]

Thus, the deferment price D expressed as a proportion of FHVP (i.e., the freehold relativity) plus the leasehold relativity must be equal to 1. Therefore:

(A1) $$D = \text{FHVP} \times e^{-qt}/\text{FHVP} \Rightarrow \; e^{-qt} = 1 - leasehold\ relativity$$

where the lease is to run for t years and the term e^{-qt} is the freehold relativity. But this value is also what it would cost to enfranchise the leaseholder. Therefore, the leaseholder or lessor wants a low value of q whilst the freeholder or lessee wants a high value of q.

The implied deferment rate q can then be obtained by taking natural log of both sides

(A2) $$-qt = ln(1 - leasehold\ relativity) \;\Rightarrow\; q = -ln(1 - leasehold\ relativity)/t$$

Using the leasehold relativity values claimed at the Tribunal gives the following table:

Table A1: Lessor vs. Lessee Implied q Rates

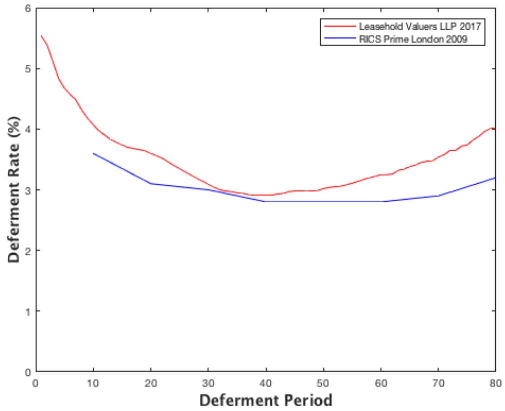
Flat #	Lessor	Lessee
	Implied q	Implied q
3	2.66%	3.81%
11	2.56%	3.80%
5	2.57%	4.04%
Average	2.60%	3.88%

[25] These deferment rates are derived from leasehold values which by definition must be net. Take void rates, for example. If you rent your property out as a series of short terms, you lose when the property is empty, but if you lease, it is let for 20, 30 years or more, where by the nature of the contract there is no void period. Likewise there are no management costs. Maintenance is trickier, however.

Focussing on the average, we obtain a lessor implied q rate of 2.60% vs. a lessee implied q rate of 3.88%.

This example is not untypical. The following chart shows deferment rates for RICS prime central London 2009 and Leasehold Valuers LLP 2017:

Figure A1: Empirical Deferment Rates



Notes: RICS = Royal Institute of Chartered Surveyors. Sources: http://www.graphsofrelativity.co.uk and Leasehold Valuers LLP.

We see that the deferment rate is typically at least 3% depending on the series and the deferment period.

These results are illustrations only, and other factors come into play. For example: (a) current leasehold values reflect the right to extend at a market value, whereas ERM borrowers have no such right (the 'lease' ends when they exit into long term care or die, and the estate has no right of extension); (b) we have assumed no marriage value, but adding marriage value would increase the implied q rates further; and (c) ordinary leaseholds tend not to terminate with the property in ruin, whereas there is evidence that very old ERM borrowers tend to neglect their property.

Appendix B: On the Validity of PRA Principle III, i.e., Why Deferment Property Values are Lower than Current Property Values

When modelling NNEGs, the deferment property value should be lower than the current property value, i.e., one would pay less for deferred possession. This idea that the deferment value is lower than the spot value is Principle III of SS 3/17.

The validity of this Principle can be demonstrated in different ways.

One demonstration goes as follows:

Let q_0, q_1, q_2, …. be the set of net rental rates for a property from now, period 0, to forever. These net rental services are the use-benefits we get from living in a property (e.g., the benefits of having a roof over our heads) or the rental incomes we could obtain by renting the property out.

Let us assume that these are all positive. After all, zero or negative rental rates do not make much sense.

Let A be the set of those net rental rates q_0, …. for periods 0 to forever.

Let B be the set of net rental rates q_t, q_{t+1} …. from periods t to forever, where $t \geq 1$.

Let C be the set of net rental rates q_0, ... q_{t-1}, for periods 0 to t-1.

Assume for the moment that the prices of A, B and C all exist.

Since the sets of rentals are positive and hence valuable, then the prices of A, B and C should each be positive. By the law of zero arbitrage, the price of A should also be equal to the sum of the prices of B and C. But since the price of C is positive, it must follow that the price of B < the price of A, i.e., the deferment price must be less than the current price and Principle III is established.

To challenge this conclusion, it is necessary to argue that some of these prices do not exist. Since the price of A is the spot price, then the price of A clearly does exist, so one would have to argue that the prices of B and/or C do not exist.

Let's note to begin with that the empirical basis of any such claim is arguable. Whilst it is manifestly obvious that the prices of B and/or C will rarely exist for some specific property, it is often possible to infer proxy prices for different types of property from comparisons of freehold and leasehold prices and it is these proxy prices that one would use for valuation purposes.[26]

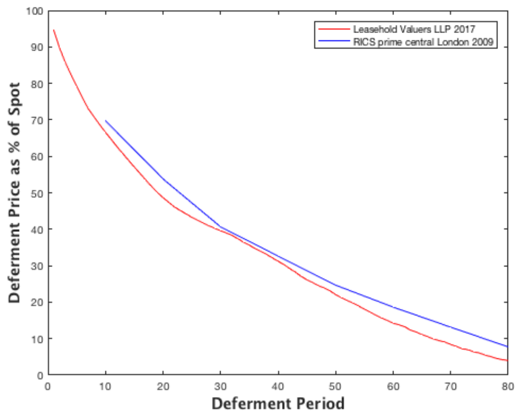
For example, consider a leasehold on a London flat with 99 years to run. The price of this leasehold would typically trade at about 95% of the price of a vacant freehold, and the

[26] Typically, one would obtain deferment prices for a particular property by applying rules of thumb to the prices for different property types. These would take account of particular features of a property such as location, parking availability, the size of the garden and so on.

corresponding freehold, i.e. the right to exclusive possession after 99 years, would trade at about 5% of the vacant value, and gives us the price of possession deferred by 99 years.[27]

Or consider the RICS and Leasehold Valuers LLP relativity graphs discussed in Appendix A. These suggest that q rates are positive for all deferment horizons going out to at least 80 years. The corresponding deferment prices are shown in Figure B1: these all fall as the deferment horizon lengthens and are always less than the spot price:

Figure B1: Empirical Deferment Prices



Notes: As per Figure A1.

But suppose for the sake of argument and contrary to the evidence just presented that some of these prices do not exist and do not have near approximations or proxies in terms of other market prices. In this situation, we simply switch the metric from prices to values and we can establish the validity of Principle III in much the same way as before. For example, if we assume that each net rental rate has a positive value, then it immediately follows that each of A, B and C has positive value, so the value of B must be less than that of A and Principle III follows. Indeed, even if we assume that the current net rental rate is positive and the others are merely non-negative, then Principle III still follows.

To challenge Principle III, one is left having to argue that net rental rates or the values of net rental rates are negative.

Let's consider possible examples.

One is where the property and the land which it stands are polluted beyond any feasible repair. Chernobyl comes to mind: even if the land could be restored to a usable state, the costs of doing so would be prohibitive. In this case, all q_0, q_1, ... are negative and will remain so. The property and the land itself would then be abandoned. This type of situation is rare, however.

A less rare case is where the property is uninhabitable and repair would be uneconomic, but the land itself is valuable. Parts of Detroit come to mind. One might then say that the (current or near current) net rental proceeds were negative, but this situation would not

[27] See the 'relativity graphs': http://www.graphsofrelativity.co.uk/.

last because the land itself is valuable. The property would be demolished, perhaps after being sold off, and the site redeveloped to restore a positive net rental stream.

A third and more common case is where the property needs repair and repair is economically feasible. The property might not generate any current net rental, but it would be repaired and a positive rental stream restored. This situation is not uncommon, but is still relatively infrequent, in that it does not apply to most properties most of the time.

The general case is that most properties most of the time generate a positive net rental stream. Therefore, when looking for a general rule to assess deferment value, the only sensible rule is to assume a positive net rental stream – and a positive net rental stream implies that the deferment value will be less than the current property value.

In short, if the prices of A, B and C all exist and are positive, then the validity of Principle III follows from zero arbitrage. If any of the prices of A, B and/or C do not exist, however, then we can still obtain Principle III by switching over to a rational valuation argument, in which it suffices to argue that the values of A, B and C are all positive because the underlying rentals have positive value.

There is also a normative argument that one can call the 'fiduciary principle'. Even where market prices do not exist, accounting principles say that the accountant *should* value economically similar assets in the same way and imply that valuation should reflect rational investor preferences. The word 'should' or 'ought' appears, e.g., in IFRS 13 B14a: "Cash flows and discount rates *should* reflect the assumptions that market participants would use when pricing the asset or liability." The fiduciary principle says that an accountant or auditor or some other person, who has an obligation of trust towards a less knowledgeable investor, must value an asset or liability as a *rational knowledgeable investor* (or 'market participant, or knowledgeable, willing independent person) would. This principle provides a safeguard against interested parties coming back along the lines of "no arbitrage doesn't apply here, so we can make up any price that benefits management, other non-fiduciaries etc." Applying this principle, the accountant, actuary etc. must at least acknowledge that rental services have positive value and this acknowledgement suffices to establish Principle III.

Guy Thomas on Principle III

To our way of thinking, the validity of Principle III should be beyond dispute, but it is not. Guy Thomas takes issue with it in a recent posting (Thomas, 2018). In this piece, he acknowledges that the loss of foregone rights (e.g., to income or use of the property) during the deferment period [i.e., the argument underlying Principle III] "appears a reasonable argument" but even so, adds that "there are also reasonable counter-arguments." As he put it:

> Housing today is owned mainly by owner-occupiers. They have a preference for a current interest to a deferred interest, because they need a roof over their heads, they like long-term security of occupation, they like being able to make their own choices on extensions and repairs, etc. In other words, they like the practical and sentimental benefits of home ownership. A minority of owners are

> buy-to-let landlords: they like understandable form of the investment, the unusual ability to finance it largely with borrowed money, and perhaps the disengagement it facilitates from the distrusted pensions and savings industry.

We would put it a little differently. Anyone who lives in a property gets the 'net rental services' of that property – the use-value benefits of a roof over their heads and so forth. Some people choose to obtain those benefits by buying their property and others by renting it. In the latter case, the property owner gets the benefit of the rent tenants pay, and in most plausible situations, the owner who rents out their property will receive a rent that more than covers the costs of maintaining their property. There are exceptions as we have explained, but these are unusual.

> For an insurer, on the other hand, these practical and sentimental benefits of a current interest in a house have no relevance. The main potential benefit of a current (as opposed to deferred) interest is the potential income from letting.

True, and this point applies to any owner who rents out their property.

> But a current interest also has several disbenefits [sic]: tenants need to be managed, houses need to be maintained, from time to time there are costs (Including possibly PR costs) of evicting tenants in arrears, and there is a possibility (through existing or new legislation) that tenants might acquire new rights.

Yes, there are costs and risks to having tenants.

> If on the other hand houses are kept vacant, this gives another set of problems: council tax, security and maintenance costs, and possibly very considerable PR costs of owning substantial amounts of empty housing.

And, yes, there are also costs from keeping properties in vacant possession.

> These disbenefits are not fanciful; their materiality can be inferred from the observable fact that despite the excellent long-term performance of housing as an investment, neither insurers nor any other financial institutions have shown any enthusiasm over the past several decades for housing as an asset class.

These passages are a roundabout way of saying that there are benefits and costs of owning property but if an owner regards the costs as outweighing the benefits, then the sensible choice for the owner is to sell. The property will then end up in the hands of an owner who *does* value the benefits as more than the costs – otherwise they wouldn't have bought the property and *someone else* would.

The lack of enthusiasm (or otherwise) of financial institutions for housing as an asset class is another question. He continues:

> So current interests in houses are evidently not attractive to insurers and other institutional investors. Deferred interest might well be more attractive, particularly if in the form of cash-settled financial contracts, so that all the

> problems of current interests are permanently avoided. Even if a deferred interest is not strictly preferred, *the relative valuation of a deferred interest compared to a current interest seems very likely to be much higher for an insurer than a typical individual owner*. (Our emphasis)
>
> Now if there were a substantial market for deferred interests, the money weight of individuals' preference for current interests versus insurers' preference for deferred interests would determine the relative *market prices* for the two types of interest (i.e. what the PRA calls the 'deferment rate'). But we have the same problem as with the hedging arguments: *the market for deferred interests does not exist on any meaningful scale*. (Our emphasis)

Leaving aside that a market for deferred interests does exist (see Appendix A), Thomas is comparing one hypothetical non-market valuation (i.e., insurers' valuations of current possession) against another (i.e., their valuations of deferred possession). A comparison of the relative valuations of spot and deferred possession made by a party that is *ex hypothesi* not a major player in the market does not establish anything about the market prices or plausible values for current possession or the market prices or plausible values for deferred possession or any relationship between them. In any case, no such comparison establishes that deferred, forward or future 'interests' have the negative value necessary to undermine the validity of Principle III.

To make the same point in a different context, suppose we value a typical stately home as being worth X times the value of a typical castle, but the market values a typical stately home as being worth Y times the value of a typical castle. But since we do not happen to own either a stately home or a castle, our views about their relative valuations are irrelevant and the only valuations that matter are those of the market.

In short, the validity of Principle III can be buttressed by sound economic theory and empirical evidence, but the counter arguments cannot.

Appendix C: The Use by Just Group of a 4.25% Expected House Price Inflation Rate in the Valuation of its NNEGs

In the text we quoted a passage from Just Group's 2016 *Annual Report* which stated that the firm had used a 4.25% HPI assumption when valuing its NNEG. When we made a presentation on equity release to the London School of Economics on 1 October 2018 equity release analyst Marcus Bernard from NUMIS suggested that we had the number up.[28]

The facts are these: the firm reported using this number in both its 2016 and 2017 *Annual Reports* (see pp. 163 and 110 respectively) and the relevance of this number in these reports is also clear, because it is the IFRS reports that shareholders would be interested in. The same number also appears in its 2018H1 results (p. 18). However in their 2017 SFCR, the firm reports an explicit q rate of 0.5% (p. 54). The latter accompanies an almost £1bn hit to their balance sheet that is offset behind transitionals (Buckner, 2018a, b).[29]

The situation for 2016 is straightforward, i.e., the firm assumes HPI = 4.25% and an implied q rate of about -2.75%.

The situation for 2017 is more involved, however.

To elaborate: in their 2017 SFCR, the firm claimed to be using a deferment rate of 0.5%:

> As at 31 December 2017, the Board considers the Matching Adjustment in the Group's balance sheet in respect of LTM notes satisfies the principles of SS3/17 giving rise to an implied property volatility of 12% and *a positive deferment rate of 0.5% on a risk neutral basis*. (2017 SFCR, p. 54)

This statement implicitly conflicts with statements in their 2017 *Annual Report* that they were using an HPI of 4.25%.

> The return on equity release assets is adjusted to allow for the risks associated with these assets – namely, the potential shortfall resulting from the No-Negative Equity Guarantee ("NNEG"). The Group calculates the shortfall in respect of the NNEG using a variant of the Black-Scholes option pricing model. Inputs required (e.g. current house prices, future house price growth and house price volatility) are derived from available market data. (2017 *Annual Report*, p. 51)
>
> In the absence of a reliable long-term forward curve for UK residential property price inflation, the Group has made an assumption about future residential

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[28] The seminar is reported in Dowd, 2018b.

[29] Page 83 of the firm's 2007 Solvency and Financial Condition report reconciles the statutory with the regulatory balance sheet. The almost £1bn figure appears as the change 'other valuation differences', from end 2016 to end 2017. However, this almost £1 billion loss does not make a significant impact on capital because it is largely offset by an increase in the PRA transitional arrangement, which is an entry on the asset side of the regulatory balance sheet that can be used to create extra regulatory capital. It is a puzzle why this latter item (which is meant to be slow-moving and declining over time) should have increased so much over one year. We find it difficult to believe that the firm would have increased this item merely to hide the hit to its capital, so there must be some other explanation that we are unaware of.

> property price inflation based upon available market and industry data. These assumptions have been derived with reference to the long-term expectation of the UK retail price inflation, "RPI", (consistent with the Bank of England inflation target) plus an allowance for the expectation of house price growth above RPI (property risk premium) less a margin for a combination of risks including property dilapidation and basis risk. An additional allowance is made for the volatility of future property prices. This results in *a single rate of future house price growth of 4.25%*, with a volatility assumption of 12% per annum. (*ibid* p.110)

The natural reading is that they are using the Black 76 formula (which takes forward prices, not spot) using an HPI of 4.25%, current house prices, volatility of 12%, etc. This reading suggests that they are taking the Black forward rate, which should be equal to risk free minus the net rental or deferment rate:

(C1) $$f = r - q$$

and replacing it (incorrectly) with the forecast HPI, i.e.,

(C2) $$HPI = r - q$$

Rearranging, we get an implied q:

(C3) $$q = r - HPI$$

If we assume that r = 1.5%, then we get

(C4) implied q = 1.5% - 4.25% = -2.75%.

The issue then is how to reconcile this incorrect implied q = -2.75% with the explicit q = 0.5% that they also claim to be using. The difference between the two is enormous.

Furthermore, the average q rate we are discussing here is a slow-moving variable, which cannot move much from one period to another. A jump of 325 basis points from -2.75% in one year to 0.5% in the next year is implausible to the point of impossible.

In any case, both q rates are way out of line with the empirical evidence set out in Appendix A.

Now for the tricky bit. In its 2018H1 results (p. 18) the firm offers the following treatment of an implied HPI vs an 'actual' or explicit HPI:

(C5) Implied HPI = actual HPI – volatility/dilapidation – effect of capital requirement – effect of securitisation = 4.25 % - 3% - 1.5% - 1.4% = -1.65%

which they round to -1.7%.

So the firm has gone from an explicit HPI = 4.25% to an implicit HPI = -1.7%!

It is, to say the least, distinctly suspicious to have an actual HPI and an implied HPI that are so far apart.

We also have to wonder why these additional terms seem to have been introduced in 2017. If it was 'correct' to use these additional terms in 2017, why did the firm not make similar adjustments in earlier years?

The derivation of the implied HPI is also problematic. The adjustment for 'volatility' is odd, given that the Black formula *already* includes an explicit treatment of volatility, namely the direct input for volatility (which the firm tells us is 12%). If the firm is using a 4.25% HPI input for the forward rate calculation and a volatility input for the put valuation, then it would be wholly incorrect to include an additional volatility 'adjustment' as well. Likewise, the deferment rate q already includes an allowance for dilapidation because the q rate is *net* of dilapidation etc. So there appears to be some obvious double counting. Likewise it is spurious to include the capital requirement, because the calculation is for the amount of capital *available*, not the capital *required*. The NNEG calculation is an input determining the amount of capital available only. The 'effect of securitisation' item is also odd and the firm gives no information on which to assess it; ergo, we have no reason to believe it.

We now rearrange (C3) as

(C6) $$r = q + HPI$$

and substitute q = 0.5% and HPI = -1.7% into (C6) to obtain

(C7) $$r = 0.5\% + -1.7\% = -\ 1.2\%!$$

So in making an explicit assumption of q = 0.5% and going from explicit HPI = 4.25% to an implicit HPI = -1.7%, the firm is also implying an astonishing r = -1.2%. Since this implied negative r rate is obviously wrong, then the analysis underpinning the derivation of its q = 0.5% calibration must also be wrong.

What seems to have happened is this: In 2016, the firm used an expected HPI rate of 4.25% to model its NNEG, equivalent to using an implicit q rate of -2.75% or thereabouts. As we have repeatedly stated, this approach is manifestly wrong, because the q rate should be much higher. In 2017, the firm *again* used the 4.25% expected HPI rate of 4.25% to model its NNEG, but this time it introduced a series of (mostly inappropriate) extra items driving a wedge this HPI rate and an implied HPI rate that is only consistent with the firm's assumed q rate of 0.5% if r = -1.2% which is clearly not the case. The derivation of the implied expected HPI rate and the corresponding 0.5% q rate is thus totally half-baked, but it appears that the firm *did* use this 0.5% q rate to value its NNEG.

Table C1 summarises the firm's NNEG valuation approaches for 2016 and 2017 in terms of their (a) explicit parameter assumptions, (b) their implied parameters and (c) their errors.

Table C1: Key Parameters of Just's NNEG Valuation: 2016 vs 2017

	2016	2017
Explicit param	HPI = 4.25%	HPI = 4.25% q = 0.5%
Implicit param	q = -2.75%	HPI = -1.7% r = -1.2%
Error	q < 0	r = -1.2%

There remains considerable uncertainty about the firm's approach to its NNEG modelling but much of this uncertainty could be cleared up if the firm were more forthcoming.

Appendix D: Does the 'Real World/'Discounted Projection' NNEG Valuation Approach Currently Meet the Financial Reporting Council's Technical Actuarial Standards?

The 'Real World'/'discounted projection' approach:

- Has never been convincingly justified by those who advocate it;
- Has not been endorsed by a recognised independent expert;
- Violates both accountancy principles and the spirit and purpose of Solvency II;
- Does not appear in the corpus of recognised scientific research journals that are subject to rigorous peer-review;
- Is contradicted by alternative approaches such as Black '76 that are used and taught all over the world and have been published in top tier academic journals, albeit that their applications are sometimes still controversial; and
- Produces results that are often absurd (e.g., negative valuations for leaseholds) and sometimes impossible (e.g., when they violate the bounds imposed by the PRA's good practice principles).

We would suggest that there is a prima facie case that this approach violates a number of the FRC's Technical Actuarial Standards (TASs):

Exhibit #1: TAS 100, para 2: "Data used in technical actuarial work shall be appropriate for the purpose of that work so that users can rely on the resulting actuarial information."

ERM actuaries have been using inappropriate data, specifically an assumed HPI rate, to obtain the forward price in the NNEG. This practice is wrong on principle.

How then can "users can rely on the resulting actuarial information"?

Exhibit #2: TAS M requires that a model is "fit for purpose both in theory and in practice."

The discounted-projection method has been known to be flawed for a long time and violates key tenets of modern pricing theory, e.g., that one should use the correct underlying variable in the option pricing equation.

Exhibit #3: Ibid 3.10 states that models must use 'neutral' or unbiased estimates, which do not incorporate adjustments to reflect the desired outcome.

Recall the PRA's statements on its website about this issue:

> Conflicts of interest is one of the key regulatory areas for the Institute and Faculty of Actuaries (IFoA) and its members …
>
> As one of the five key principles of the Actuaries' Code [of Conduct], impartiality is placed in sharp focus in the context of professional conflicts of interest, *actual or perceived* …

> Impartiality in the context of the Actuaries' Code means that: members will not allow bias, conflicts of interest, or the undue influence of others to override their professional judgement.[30] (Our emphasis)

The key phrase here is "*actual or perceived*" and one can argue that some firms *might be perceived* as have been using the discounted projection method to bolster their reported profit margins. Hosty *et al.* (2007) provides the basis for all subsequent models used by actuaries to value the NNEG and states that "For providers attempting to price the NNEG on a market consistent basis [aka the correct approach] there is insufficient product margin in order to provide a competitive product." Section 7.3.3 suggests that under a market consistent approach the product would not be profitable, whilst the discounted projection model "has produced a significantly lower cost."

Exhibit #4: TAS 200 8 states that "Measures, assumptions and judgements used to derive any estimates described as "best estimate", "central estimate" or other similar terms shall be neither optimistic nor pessimistic and shall not contain adjustments to reflect a desired outcome." See above on profitability considerations trumping technical considerations and "actual versus perceived" conflicts of interest.

Exhibit #5: TAS D 5.5 requires checks on whether data are relevant, likewise 5.6. Section 4 of PRA DP 1/16 discusses which data are a "relevant market input." Likewise, SS 3/17 para 3.18 based on elementary option pricing theory states that expectations of future property growth are not relevant as an input to obtain the forward price used to value the NNEG.

Exhibit #6: TAS D requires that data be consistent with data from other sources. However, the discounted projection model often implies negative leasehold values, which are not consistent with the observed leasehold values, which are always positive.

Exhibit #7: The Insurance TAS requires that actuarial information provided to managers and the governing body of an insurer must be relevant, and that "calculations are carried out using measures, methods and assumptions which are fit for purpose and are performed correctly."

Now maybe it could be objected that option valuation is a difficult technical subject, and that the use of the discounted projection approach might reflect disagreement about technical matters so abstruse that practitioners cannot be blamed for these errors. Any such response would be incorrect, however. The valuation principles set out in PRA SS 3/17 and elsewhere by the PRA are elementary and the challenges to those principles from the Institute are flawed in ways that should be obvious to any competent professional.

We reiterate again that these principles are based on first-principles economics and their validity is not dependent on any option pricing model or potentially questionable assumptions such as market completeness or perfect market liquidity. Nor does their validity depend on any economic or finance model or the way it might be calibrated.

[30] https://www.actuaries.org.uk/upholding-standards/conflicts-interest.

The continued widespread use of the discounted projection approach suggests that actuaries have failed to adopt developments in financial theory despite the initiatives taken in the aftermath of the Equitable fiasco (see Morris Review 4.4 and passim) and raises questions over whether current actuarial practices in the equity release sector meet required minimum standards.

They also raise broader questions about actuarial education, the roles of the Institute and the FRC and the effectiveness of current governance arrangements.

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