270.615: Inverse Modeling and Data Assimilation
Spring Semester 2025.

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Synopsis

This graduate class will introduce modern inverse modeling and data assimilation techniques. These powerful methods are used in atmospheric science, oceanography, and geophysics and are growing more widespread. Topics include: least-squares, singular value decomposition, Green's function inversions, variational data assimilation (method of Lagrange multipliers), Kalman filters, and Bayesian methods. The class will include lectures on concepts and theory, and practical experience working on computer assignments. Applications are mainly drawn from geophysical fluids, especially oceanography.

The class is a follow-on to 270.307 Geoscience Modelling. The class will be informal: as students, you will help shape and refine the material.

We'll spend about half of our time working on assignments that illustrate the theoretical ideas. We will use MATLAB software which is widely used in science and industry, but you can use Python (or Julia, or something else) if you want.

Permission of the instructor is required to attend class. Familiarity with linear algebra, statistics, classical physics, fluid dynamics, and calculus (pdes) is assumed at the level of an advanced undergraduate. Familiarity with MATLAB (or similar scientific programming software) is an advantage. Similarly, 270.307 Geoscience Modelling is an advantage, but not essential. The course may be accessible to you without these requirements, but expect to do some remedial work to catch up.

Class materials will be posted to Canvas. I plan to use Panopto to capture lectures and post to Canvas.

Classes will be taught in person.

I don't hold regular office hours, but feel free to talk to me after class or email to setup a meeting.

Learning Goals

Students who complete this course will demonstrate the ability to (assignments are in parentheses):

- Explain and use determined, over-determined, and under-determined least squares problems. (Tomography)
- Explain and use the singular-value decomposition to compute empirical orthogonal functions. (Internal Waves)
- Explain and use Green's functions to solve linear inverse problems. (Tracer Inversion)
- Explain and use the method of Lagrange multipliers to solve linear inverse problems. (Tracer Inversion)

plus (probably) one of:

- Explain and use adjoint methods to quantify system sensitivity and to solve non-linear inverse problems. (ENSO Inversion)
- Explain and use a Kalman filter to track the state of a system. (Celestial Mechanics)
- Explain and use Bayes theorem to track the state of a system. (Planetary Evolution of Life)

Assessment

Four or five extended computer assignments will be written up and turned in for credit. These assignments will require consistent work over many days and are collaborative. There will be no final exam. Attendance in person at each class is expected, and is essential for good performance. Please inform me ahead of time if you cannot attend class, or as soon as is practicable. We will adhere to the University guidelines on illness (including COVID, available at: https://covidinfo.jhu.edu/): don't worry about catching up with class until you're well again. You will not be academically penalized for following the advice of health professionals! Similarly, religious holidays are valid reasons to be excused from class, but please inform me well in advance.

Ethics

The following guidelines are taken seriously in this class:

The strength of the university depends on academic and personal integrity. In this course, you must be honest and truthful. Ethical violations include cheating on exams, plagiarism, reuse of assignments, improper use of the Internet and electronic devices, unauthorized collaboration, alteration of graded assignments, forgery and falsification, lying, facilitating academic dishonesty, and unfair competition.

In addition, the specific ethics guidelines for this course are: *In this course, collaboration on computer assignments in the classroom is encouraged. The write-ups of the computer assignments must be done individually, however, without any collaboration other than sharing of printouts of your programs and results. If you have questions about this policy, please ask the instructor.*

Report any violations you witness to the instructor. The policy can be found here: https://e-catalogue.jhu.edu/arts-sciences/full-time-residential-programs/graduate-policies/graduate-specific-po

Diversity and Inclusion

Johns Hopkins University values diversity and inclusion. We are committed to providing welcoming, equitable, and accessible educational experiences for all students. Students with disabilities (including those with psychological conditions, medical conditions and temporary disabilities) can request accommodations for this course by providing an Accommodation Letter issued by Student Disability Services (SDS). Please request accommodations for this course as early as possible to provide time for effective communication and arrangements. For further information or to start the process of requesting accommodations, please contact Student Disability Services at Homewood Campus, Shaffer Hall #101, call: 410-516-4720 and email: studentdisabilityservices@jhu.edu or visit https://studentaffairs.jhu.edu/disabilities.

Classroom Climate: I am committed to creating a classroom environment that values the diversity of experiences and perspectives that all students bring. Everyone here has the right to be treated with dignity and respect. I believe fostering an inclusive climate is important because research and my experience show that students who interact with peers who are different from themselves learn new things and experience tangible educational outcomes. Please join me in creating a welcoming and vibrant classroom climate. Note that you should expect to be challenged intellectually by me and your peers, and at times this may feel uncomfortable. Indeed, it can be helpful to be pushed sometimes in order to learn and grow. But at no time in this learning process should someone be singled out or treated unequally on the basis of any seen or unseen part of their identity. If you ever have concerns in this course about harassment, discrimination, or any unequal treatment, or if you seek accommodations or resources, I invite you to share directly with me. I promise that we will take your communication seriously and to seek mutually acceptable resolutions and accommodations. Reporting will never impact your course grade. You may also share concerns with the department chair (Ben Zaitchik, zaitchik@jhu.edu), the Director of Graduate Studies (Emmy Smith, efsmith@jhu.edu), the Assistant Dean for Diversity and Inclusion (Araceli Frias-Ohane, afrias3@jhu.edu), or the Office of Institutional Equity (oie@jhu.edu). In handling reports, people will protect your privacy as much as possible, but faculty and staff are required to officially report information for some cases (e.g. sexual harassment).

Anxiety and Mental Health

If you are struggling with anxiety, stress, depression or other mental health related concerns, please consider visiting the JHU Counseling Center. If you are concerned about a friend, please encourage that person to seek out their services. The Counseling Center is located at 3003 North Charles Street in Suite S-200 and can be reached at 410-516-8278 and online at `http://studentaffairs.jhu.edu/counselingcenter/`

Textbooks

There is no required textbook for this class. I'm writing a book on this class and 270.307 Geoscience Modeling, however, and will distribute some draft text from it. Relevant recommended texts are:

- *Wunsch* [2006] (ISBN 0521854245) covers almost all the material we'll discuss, and much more besides. It does not cover Bayesian methods, however.
- *Gauch* [2003] has good conceptual coverage of Bayesian methods and is a wonderful introduction to scientific method, including much wisdom on statistics and probability.

Wunsch [1996] is a good alternative to *Wunsch* [2006] with much overlapping material. It's more focussed on oceanographic applications, however.

Topics

We will probably cover five or six of these topics, including (probably) 1–4:

1. The canonical least-squares inverse problem. Overdetermined case; underdetermined case (tapered-weighted least squares, Gauss-Markov theorem, singular-value decomposition); geometrical interpretation. Application to tomography.
2. Empirical orthogonal functions: An illuminating basis set. Application to ocean internal waves.

3. Green's functions: Another illuminating basis set. Application to ocean transient tracer inverse problem (from *Gray and Haine* 2001).

4. Variational data assimilation: the method of Lagrange multipliers. Application to ocean transient tracer inverse problem and a simple non-linear ENSO model. Interpretation of adjoint variables as system sensitivity.

5. Sequential data assimilation: the Kalman filter. Application to a model of planetary orbits.

6. Bayesian methods. Application to evolution of life on planets.

References

Gauch, H. G. (2003), *Scientific method in practice*, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 435 pages.

Gray, S. L., and T. W. N. Haine (2001), Constraining a North Atlantic ocean general circulation model with chlorofluorocarbon observations, *J. Phys. Oceanogr.*, *31*, 1157–1181, doi:10.1175/1520-0485(2001) 031<1157:canaog>2.0.co;2.

Wunsch, C. (1996), *The ocean circulation inverse problem*, 1st ed., Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, doi:10.1017/cbo9780511629570, 442 pp.

Wunsch, C. (2006), *Discrete Inverse and State Estimation Problems*, 1st ed., Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, doi:10.1017/cbo9780511535949, 371 pp.

Schedule

We will meet twice a week for two 80 minute classes in Olin 145. Our schedule is: Tuesdays and Thursdays at 09:00–10:20. These times are somewhat flexible—please let me know if you'd like to attend class, but have a conflict. The tentative schedule is as follows:

- 21, 23 Jan, Week 1: Introduction to the class. Least squares. **Tomography**.
- 28, 30 Jan, Week 2: Least Squares continued.
- 4, 6 Feb, Week 3: Least Squares continued.
- 11, 13 Feb, Week 4: Empirical orthogonal functions. **Ocean internal waves.**
- 18, 20 Feb, Week 5: Empirical orthogonal functions continued.
- 25, 27 Feb, Week 6: Empirical orthogonal functions continued.
- 4, 6 Mar, Week 7: Green's functions. **Ocean tracer inversion.**
- 11, 13 Mar, Week 8: Green's functions continued.
- 18, 20 Mar, SPRING BREAK
- 25, 27 Mar, Week 9: Green's functions continued.
- 1, 3 Apr, Week 10: Variational data assimilation. **ENSO model.**
- 8, 10 Apr, Week 11: Variational data assimilation continued.
- 15, 17 Apr, Week 12: Variational data assimilation continued. (Instructor may be out of town this week.)
- 22, 24 Apr, Week 13: Sequential data assimilation or Bayesian methods. **(Final assignment)**.

Computer assignments are written in **bold** font. The drop deadline is 3 March.