4 Type II Supernova Physics

We consider the physics and phenomenology of supernovae, energetic explosions of stars, focussing on the so-called Type II type of supernova, which involves an evolved, massive star which is not necessarily a member of a binary system. As discussed in the next lecture, a supernova event produces a shock wave of rapidly expelled stellar material which is bombarded by a huge neutron flux. Neutron capture events result, leading to the production of elements heavier than iron and nickel, which we haven’t been able to produce in the stellar processes discussed so far. Hence, heavy elements are produced exclusively by supernovae, and the development of earth and its life forms depend on at least one cycle of heavy star evolution and subsequent supernovae.

4.1 Massive star core collapse sequence

Consider an “evolved” star, one which has evolved off the main sequence and has an evolved mass $M$ which may be significantly smaller than its zero-age main-sequence (ZAMS) mass. As discussed in previous lectures, at some point the pressure due to electron degeneracy becomes important. This is the pressure which results from electrons being forced to higher-energy states due to the Pauli exclusion principle, which applies to all fermions (spin=$\frac{1}{2}$ particles), such as electrons or neutrons. Now if $M < 1.4M_\odot$, then the electron gas is non-relativistic, and the electron pressure is $\propto n^{5/3}$, where $n$ is the electron number density, and the star is stable under gravitational collapse, since the gravitational pressure is $\propto n^{4/3}$. The fate of such a star is a white dwarf, as discussed before. The mass $1.4M_\odot$ is known as the Chandrasekhar mass.

However, if the evolved mass is $> 1.4M_\odot$, then the electron gas is relativistic, the electron pressure is $\propto n^{4/3}$, and the star is in unstable equilibrium with the gravitational pressure. Eventually, an unstable equilibrium always moves toward a stable equilibrium at with lower potential energy configuration. A massive, evolved star would eventually have a core temperature of $\sim 5 \times 10^9$ K and a density of $\sim 3 \times 10^{10}$ kg/m$^3$. The average thermal kinetic energy of nucleons is then $\sim 1$ MeV and the Boltzmann distribution extends to sufficiently high energy to allow fusion of nuclei up to nickel and iron (see Figure 1 from Lecture 4). After iron, it is energetically unfavorable to go to higher atomic number, so the massive star develops an iron core with silicon.
burning in the next sub-shell outward in radius. The silicon eventually ends up as iron, too, but after about one day this fuel is exhausted. Our massive star is now in unstable mechanical equilibrium, supported only by the electron degeneracy pressure.

The following describes what is thought to be a typical sequence of events for the collapse of a massive star, resulting in a supernova event. However, keep in mind that rotation and magnetic fields, which we have ignored so far, may have an important role, although the outcome is unlikely to change significantly.

1. After fusion processes fizzle out, there is some gravitational collapse which causes heating to $T \sim 10^{10}$ K. This is sufficient to trigger two processes:

   (i) Photo-disintegration of iron and the subsequent photo-disintegration of the products, eventually leading to complete “inverse fusion,” with products $p$ and $n$:

   $$\gamma + ^{56}\text{Fe} \leftrightarrow \ldots \rightarrow 13^4\text{He} + 4n ; \gamma + ^4\text{He} \rightarrow 2p + 2n \quad (16)$$

   (ii) Inverse beta decay. When electrons have $K > 3.7$ MeV then the following can occur:

   $$e^- + ^{56}\text{Fe} \rightarrow ^{56}\text{Mn} + \nu_e$$

   and by time we get down to nucleons:

   $$e^- + p \rightarrow n + \nu_e \quad (17)$$

   Note that the Fermi energy of the degenerate electron gas is $\approx 4$ MeV at a density of $10^{12}$ kg/m$^3$, so there are plenty of electrons which can trigger these inverse beta decays.

2. The processes above are endothermic, that is they remove kinetic energy (and hence fluid pressure) from the core. The unstable core now quickly collapses.

3. The iron core is now in free fall. The time of fall to a much smaller equilibrium radius is $\sim 100$ ms.
4. During collapse, the \textit{neutronization} processes in Eqs. 16 and 17 proceed rapidly. Neutrinos result as well, and these mostly exit the star. This neutrino emission represents about 1\% to 10\% of the total emission, the remainder resulting from subsequent steps.

5. The collapse ends when the core reaches nuclear density. Actually, the density exceeds nuclear briefly by what is estimated to be a factor of 2 to 3. This is discussed with some more detail in Section 4.3 below.

6. The core now strongly \textit{bounces} back to nuclear density from the super-nuclear density. We can think of the protons and neutrons as bags of quarks bound together by very strong springs (spring constant $\sim 10$ GeV/fm$^2$) which are compressed by the collapse, but then spring back to equilibrium, thus the bounce.

7. The bounce sends a shock wave outward at high velocity, blowing off the remaining stellar atmosphere in the process. One the shock reaches the outer atmosphere, the photons emitted by recombination, powered by the shock itself and by subsequent nuclear decays, become the visible supernova explosion.

8. The core will radiate away its huge energy content in neutrinos, as discussed below, and the remnant core will settle down into a \textit{neutron star}. The radius is something like 15 km, depending on initial core mass, but has a mass of 1.4 to about 3 $M_\odot$.

9. The neutron-rich shock, meanwhile, will induce creation of elements heavier than iron by neutron capture. This will be discussed in the next lecture in more detail.

10. The shock continues into interstellar space at speeds of $\sim c/10$. For example, the crab nebula, resulting from the 1054 A.D. supernova is large and still expanding.

11. The neutron star may become visible in radio as a \textit{pulsar}, depending on rotation and magnetic fields. The crab’s neutron star is indeed a very “loud” pulsar, faithfully producing a radio burst once per revolution, every 33.3 ms.

The visible-light luminosity of a typical supernova is roughly $10^{42}$ J, with a peak power of $10^{36}$ J/s = $10^{36}$ W. This is about a factor $10^{10}$ greater than the
solar luminosity, which is comparable to an entire galaxy. The visible light decays exponentially as unstable nuclei decay, so the supernova is visible for weeks, depending on its location. This is discussed more in the next lecture. However, as impressive as the light output is, it represents only about 1\% of the total energy output, as we shall see in Section 4.4. We follow up now with some detail.

4.2 Energy from collapse

We saw earlier that the total gravitational energy released in collapse is the change in potential energy given by

\[ E_{\text{grav}} = \frac{3GM^2}{5R} \]

when the final radius \( R \) is much smaller than the initial. (In our case, the ratio is \( \sim 10^{-3} \).) Using \( M = 1.5M_\odot \), that is, just over the Chandrasekhar limit, then with \( R = 15 \text{ km} \) we get

\[ E_{\text{grav}} = 6 \times 10^{46} \text{ J} = 4 \times 10^{59} \text{ MeV} \]  \( \text{(18)} \)

This is a big number! It will feed into the energy release of the supernova.

4.3 A very large nucleus

As mentioned above, the collapse proceeds to nuclear density, where it is halted by the strong nuclear force. It takes collision energies of \( \sim 10 \text{ GeV} \) to partially break up a nucleon (neutron or proton). As we shall see momentarily, the average energy from the collapse is still 2 orders of magnitude from this. So the nucleons, predominantly neutrons at this point, get squeezed by the collapse to sub-nuclear volume, then rebound, giving rise to the supernova bounce and the return to nuclear density.

So we are justified in treating the collapsed core as closely packed nucleons, essentially at nuclear density, and consisting primarily of neutrons. As we saw with the electrons, the neutrons will also form a degenerate “gas” since they are fermions. We return to this briefly at the end. In any case, the collapsed core will settle down into what is termed a neutron star.

Nuclear density \( \rho_N \) we can estimate by the average density of a proton (or neutron):

\[ \rho_N = m_p/(4\pi r_p^3/3) = 2 \times 10^{17} \text{ kg/m}^3, \]
where we have used \( r_0 = 1.2 \) fm for the proton radius, based on lab scattering measurements. With \( \rho_N \) as the average nuclear density, then the total mass of this proto-neutron star is

\[
M_{ns} = N m_p = \rho_N 4\pi R^3 / 3 ,
\]

where \( N \) is the total number of nucleons (neutrons). Combining these, and using \( M_{ns} = 1.5M_\odot \) yields:

\[
R = r_0 N^{1/3} = 15 \text{ km} ; \quad N = 1.5M_\odot / m_p = 2 \times 10^{57} .
\]

Combining Eqs. 19 and 18 gives an estimate for the average energy per nucleon of \((3 \times 10^{39} / (2 \times 10^{57}) \approx 100 \text{ MeV}/\text{nucleon}. \) This number provides two lessons:

- It is a large kinetic energy, but far short of what is required to break up the nucleons.
- Recall from Fig. 1 of Lecture 4 that iron had the largest binding energy per nucleon at 8 MeV/nucleon. The collapse provides more than enough energy to completely break up even iron into constituent nucleons, as we had presupposed.

### 4.4 The first seconds of the proto-neutron star

Because the proto-neutron star is so dense, a qualitatively new feature is manifest. Namely, the vast majority of the collapse energy of Eq. 18 can not easily escape. To see this, we calculate the mean free path of the most weakly interacting particles, the neutrinos.

As we saw in Lectures 5 and 6, the collision rate is given by \( n \sigma v \), where \( n \) is the number density and \( \sigma \) is the interaction cross section. Hence, the time between collisions is the inverse of this; and the mean distance between collisions (the mean free path) is \( \ell = v / (n \sigma v) = 1 / (\sigma v) \). In this case, \( n = N / (4\pi R^3 / 3) \) and \( \sigma \) is the neutrino-nucleon cross section. The so-called charged-current cross section for the process \( n + \nu_e \rightarrow p + e^- \) is given by

\[
\sigma_{cc} = \frac{5.8}{\pi} G_F^2 E^2 \approx 10^{-45} \left( \frac{E}{1 \text{ MeV}} \right)^2 \text{ m}^2 ,
\]

where \( G_F \) is the weak (Fermi) coupling constant encountered in Lecture 5. If we use \( E_\nu = 20 \) MeV, this gives \( \ell \approx 2 \) m! The mean free path for neutrinos attempting to escape the collapsed core is only a few meters.
As it turns out, the neutral current scattering process $\nu_x + n \rightarrow \nu_x + n$ is also possible here, where $x$ represents any of the 3 neutrino species. A similar calculation (a bit more complicated) for this process gives $\ell \approx 10$ m. Hence we arrive at the following conclusions:

- The collapse energy is locked in the proto-neutron star and can only be carried away by neutrinos. Estimates are that about 99% of the total energy of collapse is carried by the neutrinos.

- The neutrinos are radiated from a thin few meter thick skin on the outside of the core.

- With $\ell \sim 10$ m, the energy transport via neutrinos is a random walk, diffusion problem. We can estimate the diffusion time to be $t \sim R^2/(c\ell) \sim 1$ s. This compares to a speed of light flight time of $\sim R/c \sim 10^{-4}$ s.

- There are effectively 6 neutrino populations in thermal equilibrium with the super-hot core – 3 species, with both neutrinos and anti-neutrinos. These all share the energy transport via the neutral current process. Hence we expect the 150 MeV/nucleon to be carried away with an average energy per neutrino of $150/6 \approx 20$ MeV.

Thus, along with the visible supernova, we expect a huge neutrino flux equal to the collapse energy $\sim 10^{59}$ MeV carried by about $10^{58}$ neutrinos in all 3 species and $\nu$ plus $\bar{\nu}$ with average energy 20 MeV.

### 4.5 SN1987a

In 1987 supernova was observed at earth located in the large magellenic cloud (LMC), a small galaxy adjacent to the milky way, at a distance of about 60 kpc. Luckily, two underground proton decay experiments were taking data. These detectors can observe neutrino events using the water technique discussed in Lecture 7a. (In fact, one of the detectors, Kamiokande, later became the Super-K detector encountered in 7a.) The two detectors simultaneously observed a burst of neutrino events about 7 hours before the optical supernova was observable. The predominant detection process was

$$\bar{\nu}_e + p \rightarrow n + e^+.$$
The neutrino data is reproduced in Fig. 11.

We can now compare the observations with our expectations. The measured neutrino energy is indeed about 20 MeV. (The apparent decline with time can be used to determine neutrino mass. The limit turns out to not be better than terrestrial experiments.) The neutrinos are indeed spread out in time by $\sim 1$ s. (Again, finite neutrino mass can disperse this somewhat.) The number of detected events was 20. Only $\bar{\nu}_e$ were observed, so the total flux at earth was $20 \times 6/\sigma_d \approx 10^{10}$/cm$^2$, where $\sigma_d$ is the detection cross section times efficiency. Translating this flux to the distance of the LMC gives a total energy of $2 \times 10^{59}$ MeV, in good agreement with our expectations from the previous section.

It is interesting to note that the neutrino burst from the collapse may in fact transfer enough energy to the supernova shock front to keep it from falling back onto the collapsed core. This may, in fact, require all 3 neutrino species (although the calculations are difficult), in which case we would require the 3 species to make heavy elements (and life on earth).
4.6 Further collapse and black holes

We discussed collapse stoppage and bounce above in terms of the strong nuclear force. However, since we form a core of mostly neutrons, we might expect neutron degeneracy to also play a role. And it does. As the proto-neutron star radiates away its collapse energy we might expect it to undergo further collapse if not for the degeneracy pressure. Carrying out the same calculation we did earlier for electrons results in an equivalent Chandrasekhar mass of about $5.6M_\odot$. If the collapsing core mass exceeds this, then we might expect the collapse to proceed to a black hole. If less, then it remains a neutron star.

However, the observed transition from neutron star to black hole is closer to $3M_\odot$. We might have expected that this would not be so simple. Among the complicating factors, all difficult to calculate, but important in this regime, are:

- repulsion due to the strong nuclear force
- neutron degeneracy
- non-linear gravity; that is, strong gravitational effects, as predicted by General Relativity
- large angular velocity
- large magnetic fields

On the note of strong gravity, we finish by mentioning the Schwarzschild radius, $R_s$, predicted by GR as the “event horizon” where proper time intervals go to zero and light cannot escape. It is given by

$$R_s = 2GM/c^2.$$ 

For $M = 5M_\odot$, $R_s \approx 15$ km, comparable to the size of our neutron star, but with larger mass. Adding mass to a neutron star would result in a black hole. If the initial collapsing core had mass $\sim 5M_\odot$ or more, the collapse might proceed directly to a black hole, although many researchers believe there would be a stopping bounce followed by infall back onto the collapsed core, leading then to a black hole.