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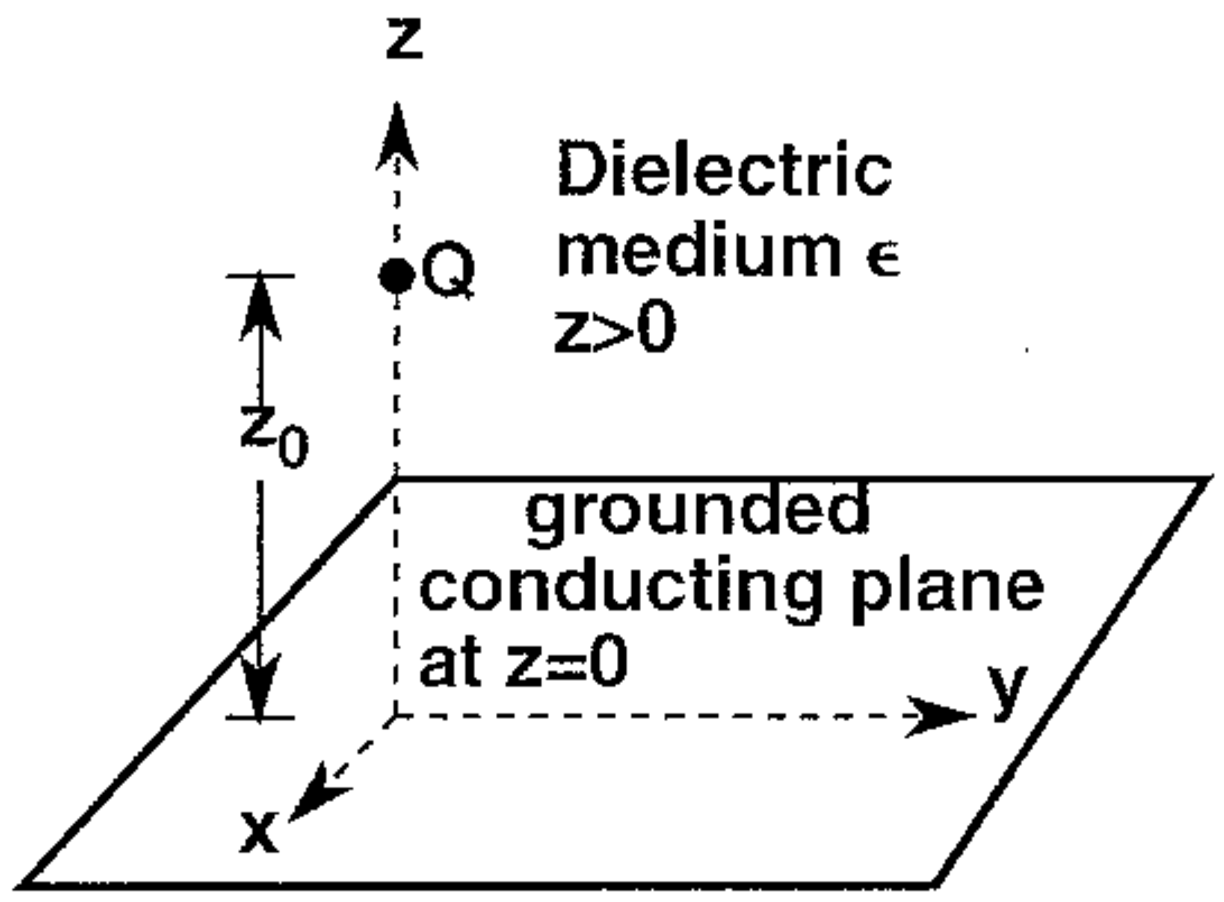
Preliminary Exam: Electricity and Magnetism

This is a closed book exam. Page 4 is a sheet of useful equations. Everything that you need is derivable from them in at most a few steps. Please attempt all five problems. Good luck!

1. A point charge Q is placed in a uniform dielectric medium of dielectric constant ϵ at the coordinate $(0,0,z_0)$ which is a distance z_0 above an infinite grounded conducting plane at $z = 0$.

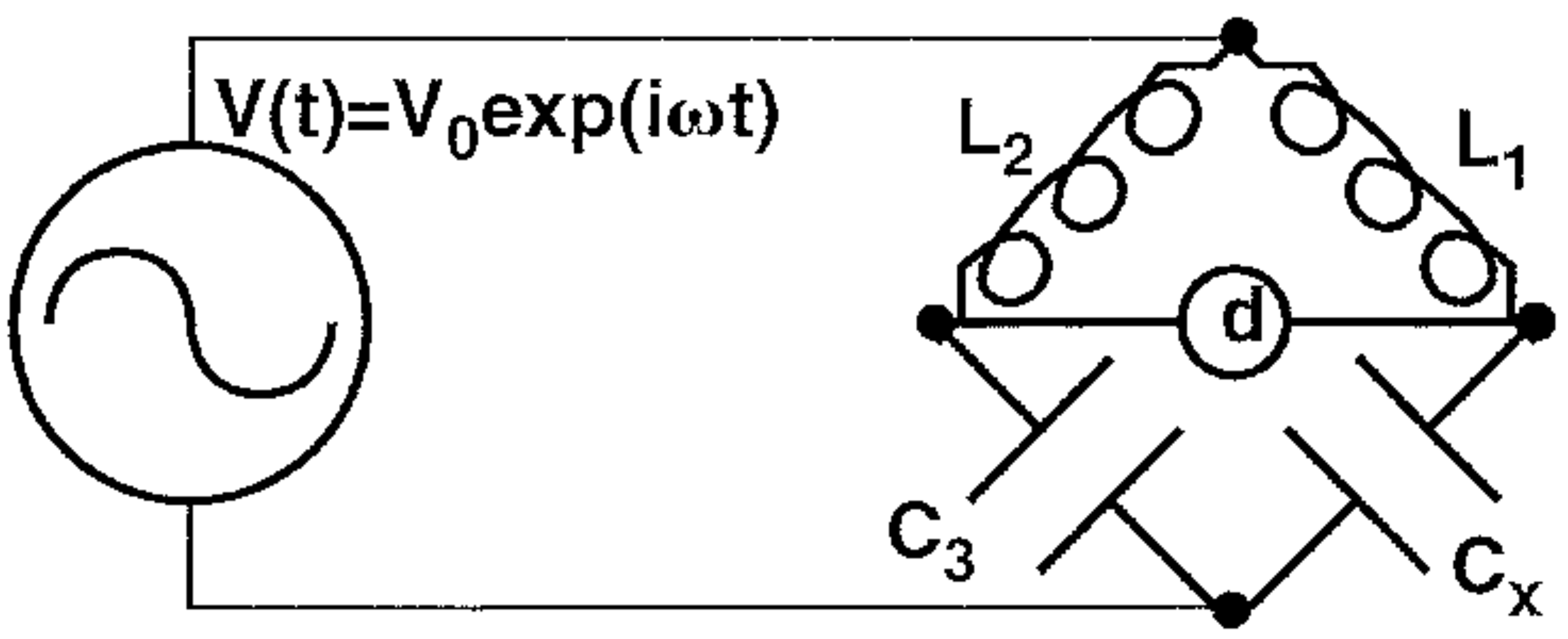


   (a) [15 pts] Calculate the surface charge density $\sigma(x, y)$ on the plane.

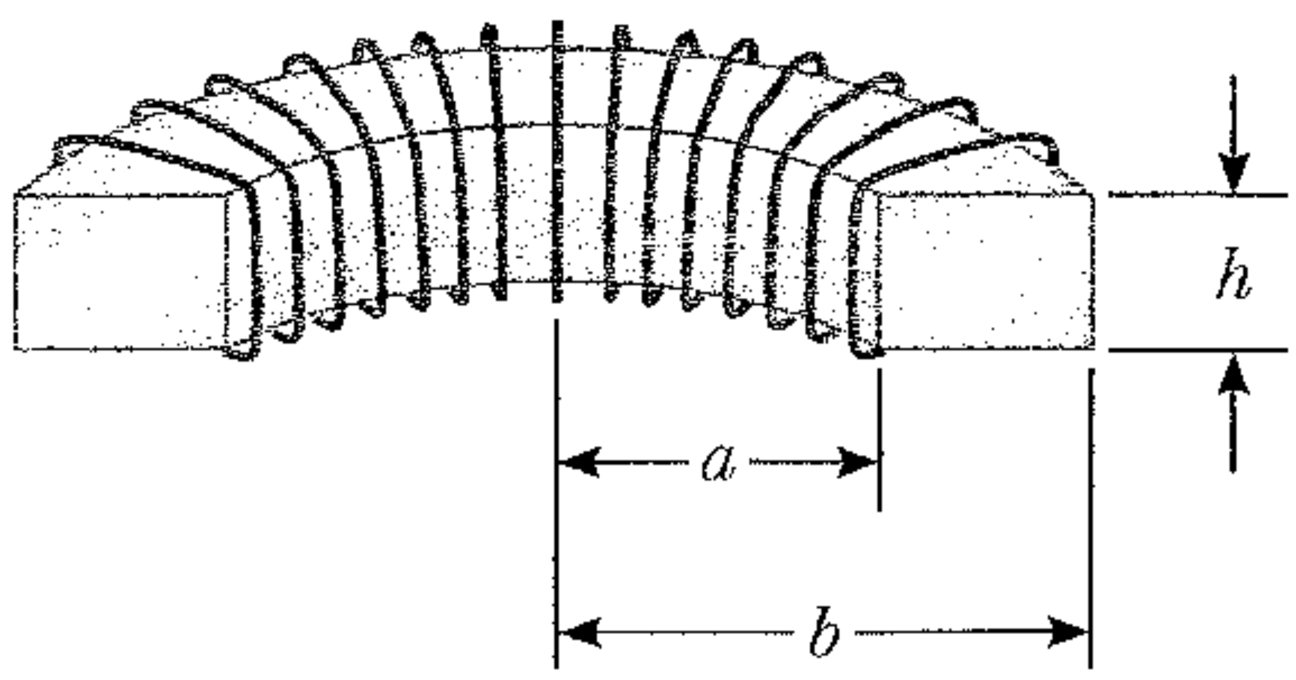
   (b) [5 pts] Calculate the direction and magnitude of the force acting on the charge.

   (c) [5 pts] What is the integrated charge on the plane in part(b)? How does it depend upon ϵ?

2. [15 pts] An AC signal is applied to the arrangement of inductors and capacitors shown below. It is found that the detector d does not detect any current flowing at all frequencies ω. What is the capacitance C_x of the unknown capacitor in terms of the known values L_1, L_2, and C_3?



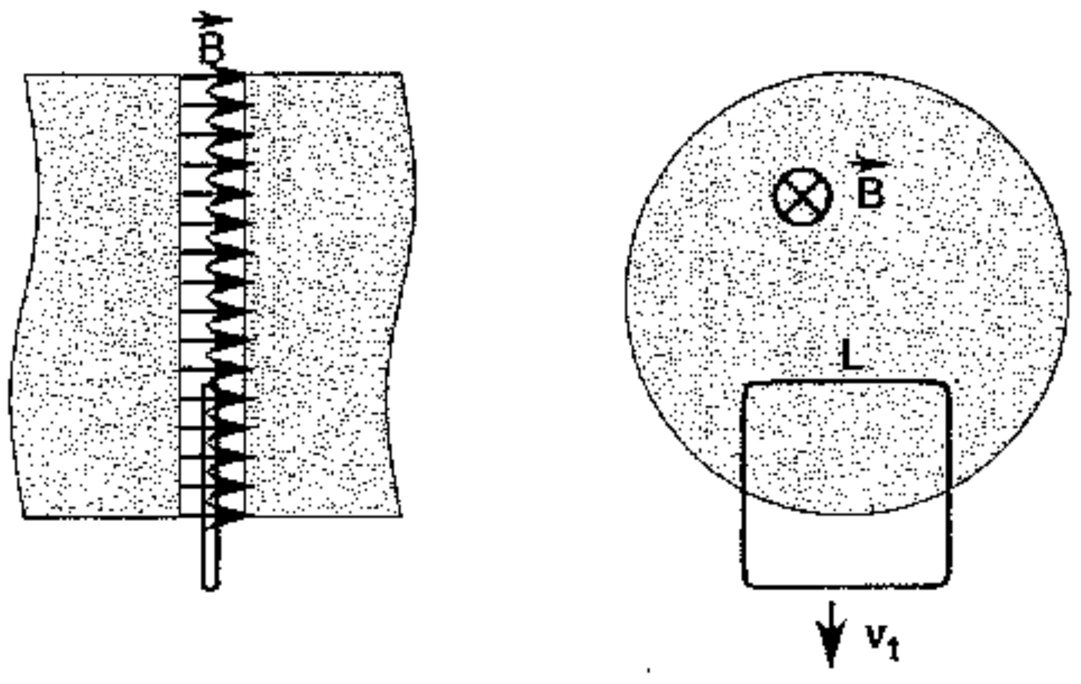
3. A toroidal coil of height h is made of N rectangular windings arranged between radii a and b (it continues in a full circle and closes upon itself).



(a) [15 pts] Calculate the inductance of the coil assuming that the turns enclose only air.

(b) [5 pts] Assuming that a constant current I is flowing through the coil, calculate the energy stored in the magnetic field.

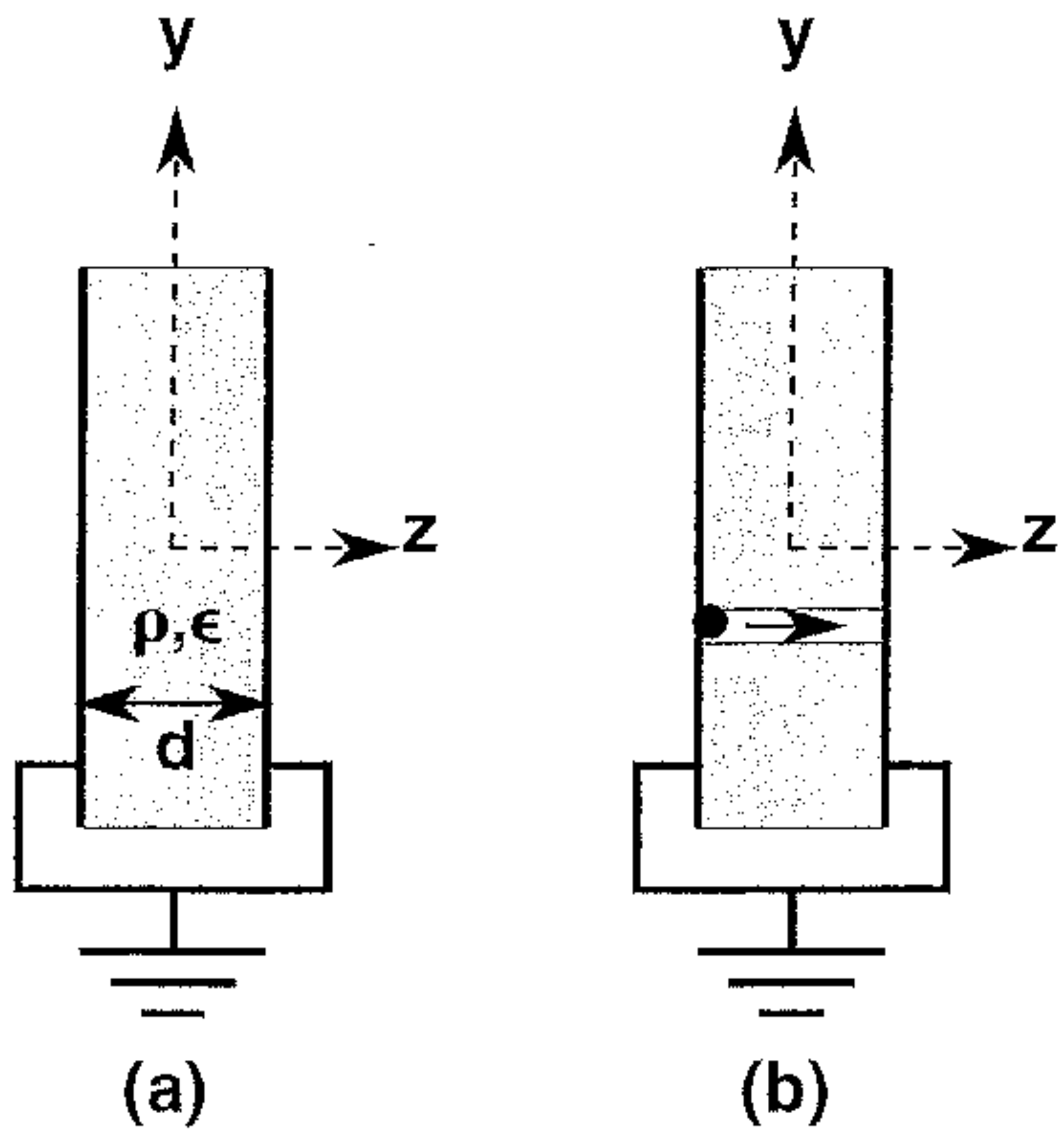
4. A square single turn coil is held at rest in the gap of a magnet. The upper part of the coil is in a region of uniform field B and the lower part of the coil is outside of the gap in air where the field is assumed to be small.



(a) [15 pts] Calculate the terminal speed v_t of the coil after it is released (assume that the top of the coil remains in the magnet gap during this time). Show that it depends only upon B, the mass density of the coil material ρ_M, and the resistivity of the coil ρ_R.

(b) [5 pts] Calculate the speed of the coil as a function of time after it is released.

5. A parallel-plate capacitor is filled with a positively charged dielectric of volume charge density ρ and dielectric constant ϵ. The plates are both grounded.



(a) [15 pts] Calculate the electric field in the gap as a function of z, the distance from the gap center. What is the sign and magnitude of the surface charge density on the conducting plates?

(b) [5 pts] A small hole is drilled in the dielectric [see figure (b)]. A negatively charged particle of unit charge and mass m is emitted at rest from the plate at $z = -d/2$. What is it's speed when it reaches the midplane at $z = 0$?

Useful Equations

- Maxwell's Equations:

$$
\begin{array}{ll}
\nabla \cdot \vec{D} = \rho(\vec{x}) & \oint \vec{D} \cdot d\vec{a} = Q_{enclosed} \\
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} & \oint \vec{E} \cdot d\vec{\ell} = -\frac{\partial \Phi_B}{\partial t} \\
\nabla \cdot \vec{B} = 0 & \oint \vec{B} \cdot d\vec{a} = 0 \\
\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} & \oint \vec{H} \cdot d\vec{\ell} = I_{enclosed} + \frac{\partial \Phi_D}{\partial t}
\end{array}
$$

- Boundary Conditions: $\hat{n}$ is normal to the plane boundary, σ and $\vec{K}$ are surface charge and current densities.

$$
\begin{array}{ll}
\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \sigma & \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0 \\
\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0 & \hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{K}
\end{array}
$$

- Static Potentials: φ and $\vec{A}$ are the electric scalar and magnetic vector potentials,

$$\vec{E} = -\nabla\varphi \quad \vec{B} = \nabla \times \vec{A}$$

- Potential of a dipole:

$$\varphi(\vec{r}) = \frac{\vec{P} \cdot \vec{r}}{4\pi\epsilon_0 r^3}$$

- Dipole Moment:

$$\vec{P} = \int \vec{x}\rho(\vec{x})d^3x$$

- Lorentz Force:

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\vec{v} \times \vec{B}$$

- Homogeneous Media:

$$
\begin{array}{l}
\vec{D} = \epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P} \\
\vec{B} = \mu\vec{H} = \mu_0\left(\vec{H} + \vec{M}\right)
\end{array}
$$

- Definitions:
  - Capacitance - $C = Q/V$
  - Resistance - $V = IR$
  - Inductance - $V = LdI/dt$