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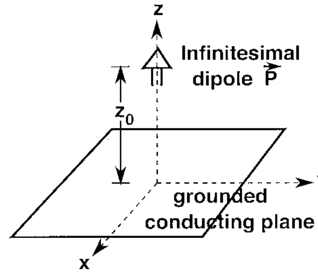
Preliminary Exam: Electricity and Magnetism

This is a closed book exam. Page 4 is a sheet of useful equations. Everything that you need is derivable from them in at most a few steps. Please attempt all five problems. Good luck!

1. An infinitesimal electric dipole is placed a distance z_0 above an infinite grounded conducting plane at the coordinate (0,0,z_0). The dipole moment $\vec{P} = P_0\hat{z}$ points orthogonally away from the plane.



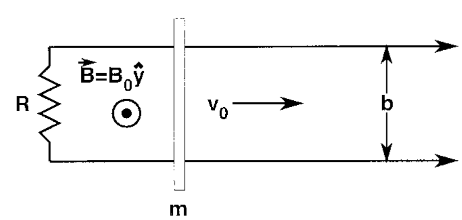
   (a) [10 pts] Calculate the surface charge density $\sigma(x, y)$ on the plane.

   (b) [10 pts] Calculate the direction and magnitude of the force acting on the dipole.

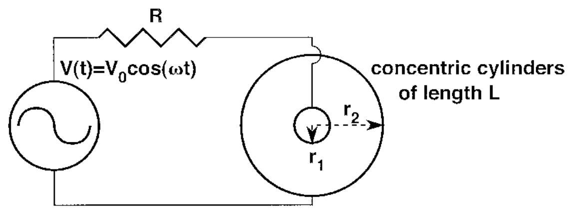
   (c) [5 pts] If the dipole is rotated by $\pi/2$ radians so that $\vec{P} = P_0\hat{x}$, calculate the surface charge density $\sigma(x, y)$.

   (d) [5 pts] What is the integrated charge on the plane in part(c)?

2. [15 pts] A pair of parallel conducting rails are separated by a distance b and are connected at one end by a resistance R. They are situated in a constant magnetic field B_0 that is perpendicular to the plane of the rails. A conducting rod of mass m spans the rails and slides frictionlessly along them. At time $t = 0$, the rod is given a velocity v_0. How far does the rod move before coming to rest?



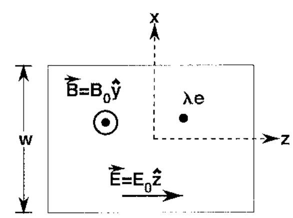
3. A pair of concentric cylinders of radii r_1 and r_2 and length L is connected to an AC voltage source via a resistance R.



(a) [7 pts] Calculate the capacitance of the cylinders assuming that end effects are negligible (eg. assume that $L \gg r_2$).

(b) [8 pts] The AC source produces a sinusoidal voltage $V(t) = V_0 \cos(\omega t)$. Calculate the stored charge on the capacitor as a function of time.

4. A charge λe is released from rest at time $t = 0$ in a slab of material ($\lambda = \pm 1$ and e is a positive number). The charge is acted upon by an electric field $\vec{E} = E_0 \hat{z}$ and a magnetic field $\vec{B} = B_0 \hat{y}$. The charge also multiply scatters as it moves through the material which produces a drag force $\vec{F}_D = -e\vec{v}/\mu$ where $\vec{v}$ is the velocity of the particle and μ is a constant called the mobility.

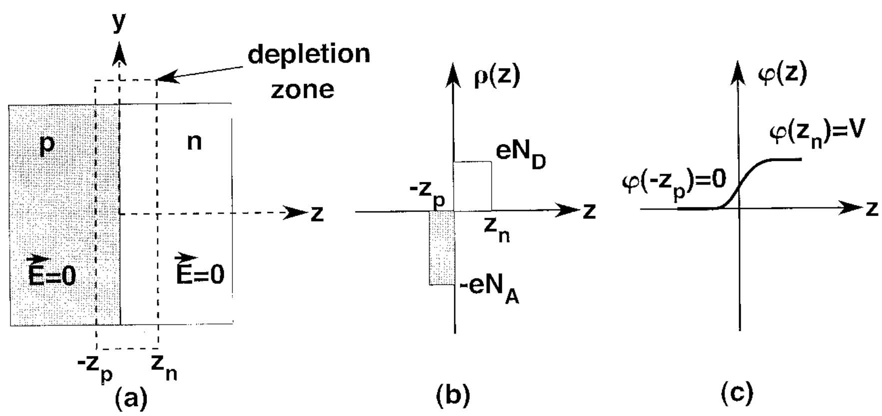


(a) [5 pts] Assuming that the magnetic field is zero ($B_0 = 0$) calculate the velocity of the particle as a function of time. What is the maximum (saturated) drift velocity? What is the characteristic time that it takes to achieve this speed?

(b) [5 pts] Assuming that the magnetic field is turned on, **ESTIMATE** the angle that the drifting particle makes with the z axis after a "long" time. Assume that the angle is relatively small (ie $v \simeq v_z$). Do not try to solve the problem exactly. This angle is called the Lorentz angle. How does it depend upon the electric field?

(c) [5 pts] Assuming that many charge carriers are flowing through a slab of finite width w, calculate [with the assumptions of part (b)] the direction and magnitude of the transverse electric field induced by the current (transverse to the z axis).

5. Two semi-infinite slabs of doped semiconductor are joined at $z = 0$ to make a pn junction. Both the p-slab (which is doped with an impurity which likes to attach conduction electrons) and the n-slab (which is doped with an impurity which likes to donate conduction electrons) are initially electrically neutral. After they are joined, some electrons diffuse across the boundary creating a non-zero electric field in a narrow region called the depletion zone [see part (a) of the figure].



A simple model for the depletion zone is shown in part (b) of the figure: between $z = -z_p$ and $z = 0$, there is a constant charge density of negatively charged electron acceptors; and between $z = 0$ and $z = z_n$, there is a constant density of positively charged electron donors,

$$\rho(z) = \begin{cases} -eN_A & -z_p < z < 0 \\ +eN_D & 0 < z < z_p \end{cases}$$

where N_A and N_D are the acceptor and donor impurity densities of the two regions. Charge conservation requires that

$$N_A z_p = N_D z_n.$$

Part (c) of the figure shows that the electric field in the depletion zone creates a voltage difference V across the zone.

(a) [15 pts] Calculate the potential function $\varphi(z)$ for any point in the depletion zone. Make sure to satisfy all boundary conditions.

(b) [10 pts] Use the result of part (a) to relate the potential difference to z_p and z_n. Solve for the total thickness of the depletion region in terms of N_A, N_D, and V.

Useful Equations

- Maxwell's Equations:

$$
\begin{array}{ll}
\nabla \cdot \vec{D} = \rho(\vec{x}) & \oint \vec{D} \cdot d\vec{a} = Q_{enclosed} \\
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} & \oint \vec{E} \cdot d\vec{\ell} = -\frac{\partial \Phi_B}{\partial t} \\
\nabla \cdot \vec{B} = 0 & \oint \vec{B} \cdot d\vec{a} = 0 \\
\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} & \oint \vec{H} \cdot d\vec{\ell} = I_{enclosed} + \frac{\partial \Phi_D}{\partial t}
\end{array}
$$

- Boundary Conditions: $\hat{n}$ is normal to the plane boundary, σ and $\vec{K}$ are surface charge and current densities.

$$
\begin{array}{ll}
\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \sigma & \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0 \\
\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0 & \hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{K}
\end{array}
$$

- Static Potentials: φ and $\vec{A}$ are the electric scalar and magnetic vector potentials,

$$\vec{E} = -\nabla\varphi \quad \vec{H} = \nabla \times \vec{A}$$

- Potential of a dipole:

$$\varphi(\vec{r}) = \frac{\vec{P} \cdot \vec{r}}{4\pi\epsilon_0 r^3}$$

- Dipole Moment:

$$\vec{P} = \int \vec{x}\rho(\vec{x})d^3x$$

- Lorentz Force:

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\vec{v} \times \vec{B}$$

- Homogeneous Media:

$$
\begin{array}{l}
\vec{D} = \epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P} \\
\vec{B} = \mu\vec{H} = \mu_0\left(\vec{H} + \vec{M}\right)
\end{array}
$$

- Definitions:
  - Capacitance - $C = Q/V$
  - Resistance - $V = IR$
  - Inductance - $V = -LdI/dt$