Preliminary Examination on Electricity and Magnetism.
September15, 2001
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There are 5 problems in the exam, each is worth 20 points to a maximum of 100 points. There are three hours available for solving them. A formula sheet is provided. No books, calculators and notes are allowed.

See Spring 2005 for Equations

Good luck!

1. A point charge q is placed at a distance R from the center of a perfectly conducting sphere of radius a, such that $R > a$. The sphere is insulated and it is electrically neutral.

   Calculate the electrostatic potential on the surface of the sphere.

2. A parallel plate capacitor is connected to a battery maintaining constant voltage V. The plates have an area A and are separated by a distance h. A dielectric of dielectric constant $\epsilon_0\kappa$ is now inserted between the plates, such that the dielectric completely fills the space between the plates.

   a) Calculate the energy stored in the capacitor before and after the dielectric is inserted between the plates.

   b) Calculate the amount of work done by the battery as the dielectric is inserted.

3. A solenoid has self inductance L and resistance R. A steady current I_0 is flowing through the solenoid. At time $t = 0$ the solenoid is shorted out.

   a) Determine the the current through the solenoid as a function of time for $t > 0$.

   b) Show explicitly that the total amount of Joule heat generated equals to the initial energy stored in the inductance.

4. A conducting circular loop of diameter D is made of a wire of diameter d, resistivity ρ and mass density ρ_m. The loop is falling from a great height h in a magnetic field with a component $B_z = B_0(1+\kappa z)$ where κ

   is some constant. Throughout its fall, the plane of the loop is parallel to the $x-y$ plane. The gravitational field is assumed to be homogeneous, the gravitational acceleration being g.

   Disregarding air resistance, find the terminal velocity of the loop.

5. A plane monochromatic electromagnetic wave is propagating in an infinite medium of conductivity σ. The medium has negligible magnetic or electric polarizability, *i.e.* $\epsilon \approx \epsilon_0, \mu \approx \mu_0$.

   Using Maxwell's equations and Ohm's law,

   a) derive the wave equation obeyed by the electric field;

   b) show that the amplitude of the wave decreases exponentially in the direction of propagation. Find the characteristic distance over which the amplitude decreases by a factor of 1/e.