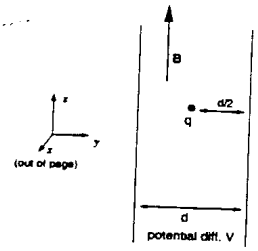
October 13, 1997

Electricity and Magnetism Preliminary Exam – Fall 1997

Do all six problems and show your work clearly. Each problem is worth 25 points. Good luck!

1. A permanent magnet in the shape of a solid sphere of radius R has a constant magnetization M throughout the interior. Find and sketch B and H everywhere.

2. A particle of mass m and charge q is released from rest midway between the two plates of a parallel-plate capacitor of separation d. There is a potential difference V between the two plates. In addition, there is a uniform magnetic field B in the direction parallel to the plates, as shown in the figure below.



(a) (10 points) Write down the equations of motion for the particle.

(b) (10 points) Solve these equations to determine the motion of the particle.

(c) (5 points) For what values of V does the particle hit the plates of the capacitor?

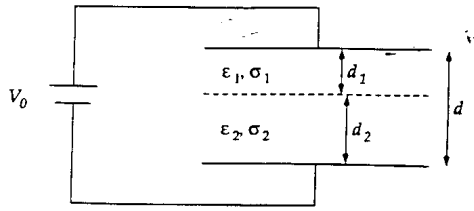
3. A surface electric charge density $\sigma = \sigma_0 \cos\theta$ is fixed to a spherical surface of radius R, where σ_0 is a constant and θ is the angle with respect to the z axis through the center of the sphere.

(a) (15 points) Find the potential everywhere.

(b) (10 points) If the spherical surface with the fixed charge spins around the z-axis with an angular frequency ω, find the magnetic field B at the center of the sphere.

4. A particle of mass m and charge q is attached to a massless, frictionless spring with spring constant k. The spring is stretched a distance A from its equilibrium position and then released. How much energy is lost per cycle due to radiation? Assume that the energy loss is small enough that the motion remains simple harmonic.

5. A parallel-plate capacitor with circular plates of radius a and separation d is filled with two layers of materials with different dielectric constants (ϵ_1 and ϵ_2) and conductivities (σ_1 and σ_2), as in the figure. The capacitor is connected to a battery of emf V_0.

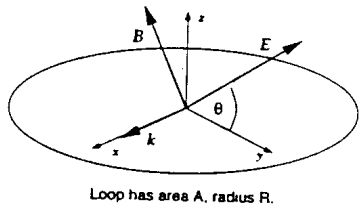


(a) (10 points) Find the electric field in each of the two materials and the current density that flows through the capacitor.

(b) (8 points) Find the total surface charge density and the free surface charge density at the interface between the two materials.

(c) (7 points) Find the magnetic field $\mathbf{B}$ within the dielectric materials.

6. A plane electromagnetic wave with circular frequency ω (wavelength λ) propagates in free space with a wave vector $\mathbf{k} = k\hat{i}$. The wave has a peak electric field amplitude of $\mathbf{E}_0$. The wave is incident on an N-turn circular receiving loop of area A oriented such that it lies in the x-y plane and forms an angle θ with the electric field vector (see figure below).



Loop has area A, radius R.

(a) (5 points) What is the wave's peak magnetic field amplitude, $\mathbf{B}_0$?

(b) (10 points) What is the magnetic flux through the loop? Assume that $\lambda \gg R$, where R is the radius of the loop.

(c) (10 points) Calculate the peak emf induced in the loop.