Davidsen/Lee

Electricity and Magnetism
Preliminary Exam. (Sept.3, 1992)
Do all 8 problems.

1. A coaxial cable consists of an infinitely long straight thin wire of radius a surrounded by a thin cylindrical conducting shell of radius b. The two conductors carry equal but opposite currents, I.

   a) Find the ***B*** field (magnitude and direction) everywhere in space.

   b) What is the magnetic vector potential ***A*** (magnitude and direction)for this system?

2. A dielectric sphere of radius R has a permanent polarization given by $\boldsymbol{P} = A\boldsymbol{r}$, where A is a constant, and r is measured from the center of the sphere.

   a) Find the volume polarization charge density, ρ_p, and the surface polarization charge density, σ_p.

   b) Find the fields **E** and **D** everywhere in space.

   c) Show *explicitly* that the total polarization charge is zero.

3. A series combination of resistance R and inductance L is put in parallel with a series combination of resistance R and capacitance C, as shown in the diagram.

   For what value of R (in terms of L and C) will the impedance be independent of frequency?

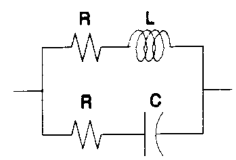

Fig. for Probem 3

4. A plastic (nonconducting) wheel with moment of inertia I has n positive electric charges q embedded on it at a radius a as in the figure. A uniform magnetic field **B** is now turned on, parallel to the axis of the wheel. Indicate the direction of rotation, and calculate the angular velocity.

5. Consider a permanent magnet in the shape of a solid sphere of radius R. The magnetization is constant throughout the interior of the magnet. Find **B** and **H** everywhere. Sketch the lines of force for **B** and **H** everywhere denoting the salient features.

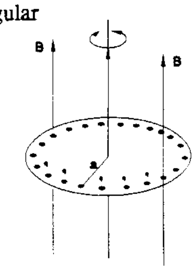

Fig. for Problem 4

6. Find the expression of the **E** field of a right-handed circularly polarized plane wave of wavelength 670 nm and power density of 1 Watt/m^2 propagating in the $+z$ direction in free space.

7. An infinitely long conducting circular cylinder is cut into two halves lengthwise, and the two halves are held at potential V_o and $-V_o$. Find the potential everywhere inside.

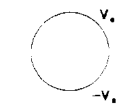

Fig. for Problem 7.

8. *Atmospheric electricity*. Near the earth's surface there is a fair-weather field of E = 100 V/m directed downward. The conductivity g(z)=J/E of air increases with height (z in m) according to

$$g(z) = [3.0 + (0.5x10^{-6}) \cdot z^2]\, x10^{-14} (\ \cdot m)^{-1}$$

(a) Considering the earth surface to be a good conductor, calculate the charge density induced on the earth's surface and the current density which flows downward owing to the fair-weather electric field.

(b)Find the potential difference between the surface and an altitude where the conductivity is much larger that the surface value. You can assume the earth's surface to be locally flat and the current density to be uniform.

See attached sheets for vector relations.
Useful information:

$c = 3 \times 10^8$ m/sec
$\mu_o = 4\pi \times 10^{-7}$ N/A^2
nm = 10^{-9} m

$$\int_0^\infty \frac{a\,dx}{a^2+x^2} = \frac{\pi}{2} \quad if\ a>0$$

$$\int_0^\infty \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}$$

VECTOR FORMULAS

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \tag{1-29}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) \tag{1-30}$$

$$\nabla \times \nabla u = 0 \tag{1-48}$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0 \tag{1-49}$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}) \tag{1-106}$$

$$\frac{d}{d\sigma}(u\mathbf{A}) = \frac{du}{d\sigma}\mathbf{A} + u\frac{d\mathbf{A}}{d\sigma} \tag{1-107}$$

$$\frac{d}{d\sigma}(\mathbf{A} \cdot \mathbf{B}) = \frac{d\mathbf{A}}{d\sigma} \cdot \mathbf{B} + \mathbf{A} \quad \frac{d\mathbf{B}}{d\sigma} \tag{1-108}$$

$$\frac{d}{d\sigma}(\mathbf{A} \times \mathbf{B}) = \frac{d\mathbf{A}}{d\sigma} \times \mathbf{B} + \mathbf{A} \times \frac{d\mathbf{B}}{d\sigma} \tag{1-109}$$

$$\nabla(u + v) = \nabla u + \nabla v \tag{1-110}$$

$$\nabla(uv) = u\nabla v + v\nabla u \tag{1-111}$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{B} \times (\nabla \times \mathbf{A}) + \mathbf{A} \times (\nabla \times \mathbf{B}) + (\mathbf{B} \cdot \nabla)\mathbf{A} + (\mathbf{A} \cdot \nabla)\mathbf{B} \tag{1-112}$$

$$\nabla(\mathbf{C} \cdot \mathbf{r}) = \mathbf{C} \qquad \text{where } \mathbf{C} = \text{const.} \tag{1-113}$$

$$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B} \tag{1-114}$$

$$\nabla \cdot (u\mathbf{A}) = \mathbf{A} \cdot (\nabla u) + u(\nabla \cdot \mathbf{A}) \tag{1-115}$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \tag{1-116}$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B} \tag{1-117}$$

$$\nabla \times (u\mathbf{A}) = (\nabla u) \times \mathbf{A} + u(\nabla \times \mathbf{A}) \tag{1-118}$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = (\nabla \cdot \mathbf{B})\mathbf{A} - (\nabla \cdot \mathbf{A})\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} \tag{1-119}$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A} \tag{1-120}$$

where

$$\begin{aligned}(\mathbf{A} \cdot \nabla)\mathbf{B} = \hat{\mathbf{x}}&\left(A_x\frac{\partial B_x}{\partial x} + A_y\frac{\partial B_x}{\partial y} + A_z\frac{\partial B_x}{\partial z}\right) \\ +\hat{\mathbf{y}}&\left(A_x\frac{\partial B_y}{\partial x} + A_y\frac{\partial B_y}{\partial y} + A_z\frac{\partial B_y}{\partial z}\right) \\ +\hat{\mathbf{z}}&\left(A_x\frac{\partial B_z}{\partial x} + A_y\frac{\partial B_z}{\partial y} + A_z\frac{\partial B_z}{\partial z}\right)\end{aligned} \tag{1-121}$$

VECTOR OPERATIONS

RECTANGULAR COORDINATES

$$\nabla u = \hat{\mathbf{x}}\frac{\partial u}{\partial x} + \hat{\mathbf{y}}\frac{\partial u}{\partial y} + \hat{\mathbf{z}}\frac{\partial u}{\partial z} \tag{1-37}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{1-42}$$

$$\nabla \times \mathbf{A} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \tag{1-43}$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \tag{1-46}$$

CYLINDRICAL COORDINATES

$$\nabla u = \hat{\boldsymbol{\rho}}\frac{\partial u}{\partial \rho} + \hat{\boldsymbol{\varphi}}\frac{1}{\rho}\frac{\partial u}{\partial \varphi} + \hat{\mathbf{z}}\frac{\partial u}{\partial z} \tag{1-85}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z} \tag{1-87}$$

$$\nabla \times \mathbf{A} = \hat{\boldsymbol{\rho}}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \hat{\boldsymbol{\varphi}}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{\mathbf{z}}\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\varphi) - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \varphi}\right] \tag{1-88}$$

$$\nabla^2 u = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial u}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial z^2} \tag{1-89}$$

SPHERICAL COORDINATES

$$\nabla u = \hat{\mathbf{r}}\frac{\partial u}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial u}{\partial \theta} + \hat{\boldsymbol{\varphi}}\frac{1}{r\sin\theta}\frac{\partial u}{\partial \varphi} \tag{1-101}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi} \tag{1-103}$$

$$\begin{aligned}\nabla \times \mathbf{A} = {} & \frac{\hat{\mathbf{r}}}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta A_\varphi) - \frac{\partial A_\theta}{\partial \varphi}\right] + \frac{\hat{\boldsymbol{\theta}}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(r A_\varphi)\right] \\ & + \frac{\hat{\boldsymbol{\varphi}}}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right]\end{aligned} \tag{1-104}$$

$$\nabla^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial \varphi^2} \tag{1-105}$$