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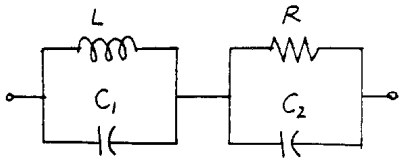
Electricity and Magnetism — January 18, 1990

Preliminary Exam

The time limit for this exam is 3 hours. Answer all 10 questions. Please start each problem on a new page and show all work.

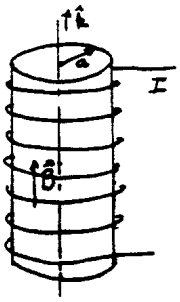
1. (10 points) In the network shown in the figure, L is a pure inductance and C_1 and C_2 are pure capacitances.



a) For what value of ω does the network appear to be a pure resistance?

b) What happens if $L > R^2C_2$?

2. (15 points) A long uniformly charged nonconducting cylinder of radius a carries a charge per unit length of λ. It is wound with N turns per unit length of wire carrying a current I. The current gives rise to a magnetic field which we assume is uniform throughout the cylinder.

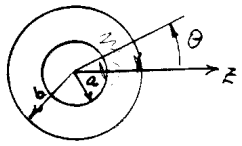


a) Find the Poynting vector $\vec{S}$ as a function of distance from the axis of the cylinder. In which direction does it point?

b) The momentum density of the electromagnetic field is given by $\vec{S}/c^2$. Find the angular momentum per unit length of the electromagnetic field about the axis of the cylinder.

c) If the current is turned off, the induced electric field exerts a torque on the cylinder. Find this torque per unit length in terms of dI/dt.

3. (13 points) Two concentric spherical shells have inner and outer radii a and b, respectively. At r = a, the potential is given by $\phi = Q\cos^2\theta$ and at r = b, $\phi = 0$. Here Q is a constant and θ is the angle with respect to an axis through the center as shown.



Find the potential in the space between the shells.

4. (8 points) Estimate the values of the following quantities:

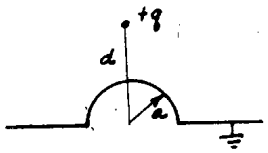
a) The conductivity of copper wire.

b) The dielectric constant of water at optical frequencies.

c) The amplitude of the electric field at the antenna of an FM radio.

d) The largest magnetic fields available in the laboratory by conventional techniques.

5. (10 points) A conducting plate has a hemispherical boss of radius a with its center on the plate as shown in the diagram. The plate is grounded and a point charge +q is brought near to it at a distance d > a, on a line normal to the plate passing through the center of the boss.



a) Find the image charge (or charges) needed to replace the plate.

b) Find the force between the charge and the plate.

[Hint: Recall image of point charge in sphere: $q' = -qa/d$; $b = a^2/d$]

6. (12 points) A dielectric sphere of radius R has a permanent polarization given by $\mathbf{P} = A\mathbf{r}$, where A is a constant, and r is measured from the center of the sphere.

a) Find the volume polarization charge density, ρ_p, and the surface polarization charge density, σ_p.

b) Find the fields E and D everywhere in space.

c) Show <u>explicitly</u> that the total polarization charge is zero.

7. (10 points) A proton (mass = 1.67×10^{-27} kg, charge = 1.6×10^{-19} C) with a velocity of 10^7 m/s is projected at right angles to a uniform magnetic induction field of 0.1 T.

a) How much is the particle path deflected from a straight line after it has traversed a distance of 1 cm?

b) How long does it take the proton to traverse a 90° arc?

8. (8 points) A cylindrically symmetric electric field is given by:

$$E_r = \frac{1}{2}\alpha\left(r - \frac{a^2}{r}\right), \qquad a \leq r \leq b;$$

$$E_r = \frac{1\alpha}{2r}(b^2 - a^2), \qquad r > b; \qquad \text{where } \alpha, a \text{ and } b \text{ are constants.}$$

Find the charge density everywhere in space ($r > a$) which produces this field.

$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

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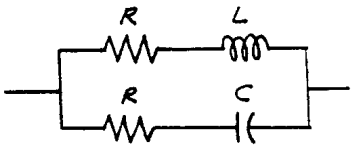
9. (10 points) A series combination of resistance R and inductance L is put in parallel with a series combination of resistance R and capacitance C, as shown in the diagram.



For what value of R (in terms of L and C) will the impedance be independent of frequency?

10. (10 points) A spherical capacitor consists of two concentric, spherical, conducting shells of radii r_1 and r_2. The region between the two shells is filled with a medium of dielectric constant K. A charge Q is placed on the capacitor.

Find the total energy, U_t stored in the dielectric material and from this determine the capacitance of the system.

$U = q\phi = \frac{1}{2}C\phi^2$ ~~$U = \frac{1}{C}\frac{q^2}{2} = \frac{1}{2}q\phi$~~

$U = \frac{1}{2}q\,\Delta\phi = \frac{Q^2}{2C}$

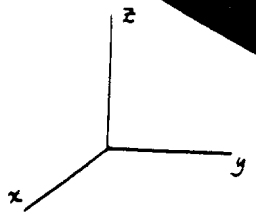
$E =$

$\Delta\phi = \int E\,ds$

$\frac{1}{2}C\Delta\phi^2$

ALTERNATE

4. (8 points) Electrons are emitted with zero velocity at $(x,y,z) = (0,0,0)$. Electric and magnetic fields exist everywhere in space given by:



$$\vec{E} = -E_z\hat{k} \quad \text{and} \quad \vec{B} = B_x\hat{i},$$

where both E_z and B_x are positive quantities.

Write down the differential equations describing the motion of the electrons.

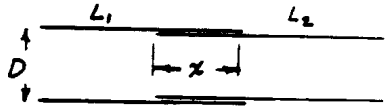
5. (10 points)

a) Suppose magnetic monopoles exist and are described by a charge density ρ^*. Derive the equivalent of Gauss' Law for magnetic monopoles.

$\nabla \cdot B = \rho^*$

b) A recent experiment seeks to detect magnetic monopoles by looking for current pulses in a superconducting coil. Explain.

6. (10 points) Consider two ideal solenoids of lengths L_1 and L_2 and of essentially the same diameter D so that they can slide freely over each other. Both solenoids have the same number of turns per unit length (n) but different currents I_1 and I_2 flowing in the same direction.



Calculate the force (magnitude and direction) between them when they overlap by a distance x.

7. (8 points) Find the charge distribution which gives rise to a potential of the form $\phi(r) = Ce^{-\alpha r}$, where C and α are positive constants.

8. (8 points) A permanent magnet has the shape of a right circular cylinder of length L. If the magnetization $\vec{M}$ is uniform and has the direction of the cylinder axis, find the magnetization current densities, $\vec{J}_M$ and $\vec{j}_M$.

Show the directions of $\vec{J}_M$ and $\vec{j}_M$ in a diagram.