PRELIMINARY WRITTEN EXAMINATIONS
ELECTRICITY AND MAGNETISM

January 13, 1983 9:00-12:00 Noon

Answer all eight (8) problems.

1a. By choice of an appropriate coordinate system, any symmetric second-rank tensor may be diagonalized. Associated with a static electric field $\underline{E}$ is the Maxwell Stress Tensor,

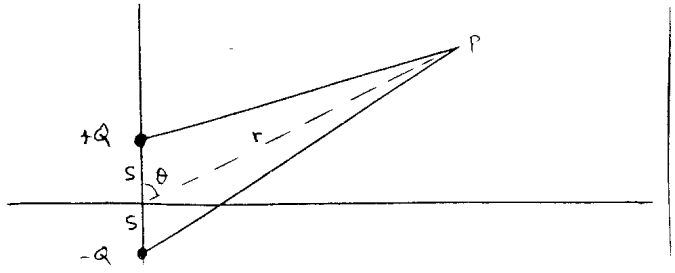
$$\epsilon_0 \begin{Bmatrix} \frac{1}{2}(E_x^2 - E_y^2 - E_z^2) & E_xE_y & E_xE_z \\ E_xE_y & \frac{1}{2}(E_y^2 - E_z^2 - E_x^2) & E_yE_z \\ E_xE_z & E_yE_z & \frac{1}{2}(E_z^2 - E_x^2 - E_y^2) \end{Bmatrix}$$

Locate a coordinate system within which this tensor is diagonal and show the diagonal form. Give a physical interpretation of the result. (Hint: Start by drawing a diagram of the situation. This problem is easy.)

1b. State and prove Gauss's Divergence Theorem. Does a similar theorem hold for a second-rank tensor? If so, state and discuss.

2. Prove that $\int_C \underline{r}\times d\underline{s} = 2\int_S \underline{n}\, da$, and use this result to discuss the magnetic moment of a filamentary stationary current loop.

3. Calculate the potential for a finite dipole exactly, and identify the quadrupole and octopole terms. (Recall that $(1+x)^{-\frac{1}{2}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} - \ldots \quad (-1<x<1)$).



4. A dielectric sphere of radius R contains a uniform density of free charge ρ_f. Calculate the potential at the center.

5. A thin conducting disk of thickness h, diameter D, and conductivity σ is placed in a uniform alternating magnetic field $B = B_0 \sin\omega t$ parallel to the axis of the disk.

   a) Find the induced current density as a function of distance from the axis of the disk.

   b) What is the direction of this current at t=0?

6. Show how Laplace's Equation can, sometimes, be solved by the method of separation of variables. Discuss.

7. A dielectric sphere of radius R is polarized so that

   $\underline{P} = (\frac{K}{r})\ \hat{\underline{r}}$, $\hat{r}$ being the unit radial vector.

   a) Calculate the volume and surface density of bound charge.

   b) Calculate the volume density of free charge.

   c) Calculate the potential inside and outside the sphere.

   d) Sketch a curve of potential versus distance from

      r = 0 to r = ∞

8. Two infinite parallel plates separated by a distance S are at potentials 0 and V_0.

   a) Use Poisson's Equation to find the potential V in the region between the plates where the space charge density is $\rho=\rho_0(x/s)$. The distance x is measured from the plate at zero potential.

   b) What are the charge densities on the plates?