

PRELIMINARY WRITTEN EXAMINATIONS
ELECTRICITY AND MAGNETISM

January 13, 1983 9:00-12:00 Noon

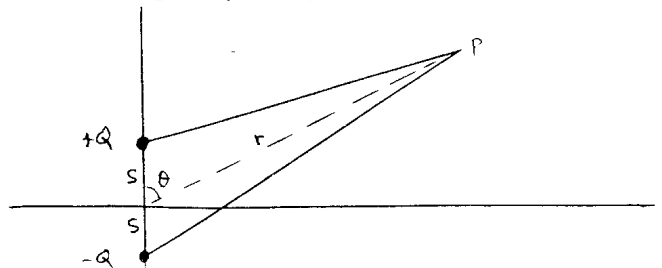
Answer all eight (8) problems.

- 1a. By choice of an appropriate coordinate system, any symmetric second-rank tensor may be diagonalized. Associated with a static electric field \underline{E} is the Maxwell Stress Tensor,

$$\epsilon_0 \left\{ \begin{array}{ccc} \frac{1}{2}(E_x^2 - E_y^2 - E_z^2) & E_x E_y & E_x E_z \\ E_x E_y & \frac{1}{2}(E_y^2 - E_z^2 - E_x^2) & E_y E_z \\ E_x E_z & E_y E_z & \frac{1}{2}(E_z^2 - E_x^2 - E_y^2) \end{array} \right\}$$

Locate a coordinate system within which this tensor is diagonal and show the diagonal form. Give a physical interpretation of the result. (Hint: Start by drawing a diagram of the situation. This problem is easy.)

- 1b. State and prove Gauss's Divergence Theorem. Does a similar theorem hold for a second-rank tensor? If so, state and discuss.
2. Prove that $\oint_C \underline{r} \times d\underline{s} = 2 \int_S \underline{n} \, da$, and use this result to discuss the magnetic moment of a filamentary stationary current loop.
3. Calculate the potential for a finite dipole exactly, and identify the quadrupole and octopole terms. (Recall that $(1+x)^{-1/2} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} - \dots$ ($-1 < x < 1$)).



4. A dielectric sphere of radius R contains a uniform density of free charge ρ_f . Calculate the potential at the center.
5. A thin conducting disk of thickness h , diameter D , and conductivity σ is placed in a uniform alternating magnetic field $B = B_0 \sin \omega t$ parallel to the axis of the disk.
- Find the induced current density as a function of distance from the axis of the disk.
 - What is the direction of this current at $t=0$?

6. Show how Laplace's Equation can, sometimes, be solved by the method of separation of variables. Discuss.

7. A dielectric sphere of radius R is polarized so that

$$\underline{P} = \left(\frac{K}{T}\right) \hat{r} \quad , \quad \hat{r} \text{ being the unit radial vector.}$$

- Calculate the volume and surface density of bound charge.
- Calculate the volume density of free charge.
- Calculate the potential inside and outside the sphere.
- Sketch a curve of potential versus distance from

$$r = 0 \text{ to } r = \infty$$

8. Two infinite parallel plates separated by a distance S are at potentials 0 and V_0 .
- Use Poisson's Equation to find the potential V in the region between the plates where the space charge density is $\rho = \rho_0 (x/S)$. The distance x is measured from the plate at zero potential.
 - What are the charge densities on the plates?