QFT Problem Set 1 - Due Feb. 12

You should read chapter 14. As usual, *problems* are extra credit seekers, although everyone should look at them.

1. **Book Problems** 14.1, 14.3, 14.4, 14.5

2. **Statistical Mechanics from the Path Integral** Define a partition function for a harmonic oscillator with Hamiltonian

$$H = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega^2 x^2 \tag{1}$$

and compute this partition function using the path integral. You'll want to introduce a Fourier decomposition of $x(t)$ with period β, ie

$$x(t) = \sum_n x_n \frac{1}{\sqrt{\beta}} e^{2\pi i n t/\beta} \tag{2}$$

The exact dependence on β for the partition function is a bit subtle since the measure of the PI depends on β when we discretize, but the dependence on ω should be unambiguous. Show that up to a (possibly divergent, β dependent) constant, the PI reproduces the partition function that you'd expect. You might need the relation

$$\sinh z = z \cdot \prod_{n=1}^{\infty} \left(1 + \frac{z^2}{(n\pi)^2}\right) \tag{3}$$

Now generalize this construction to free field theory. You should see that the formal answer is

$$\sqrt{\det(\partial^2 + m^2)} \tag{4}$$

where ∂^2 acts on the space of functions periodic in Euclidean time with period β (but arbitrary in the 3 spatial directions). You should be able to compute the partition function for relativistic scalar particles (or at least its dependence on m) using this formalism.

*Generalize this to Grassman variables $\psi(t)$ and $\bar{\psi}(t)$ with

$$H = \bar{\psi}\dot{\psi} + \omega\bar{\psi}\psi \tag{5}$$

and the antiperiodic boundary condition $\psi(t+\beta) = -\psi(t)$ (why does that make sense?). You should get the partition function of a two state system, ie a system with one fermionic degree of freedom.*

Generalize this to the photon field, with the usual action and gauge fixing function. You should find the correct statistical result, including the correct number of photon polarizations.