QFT Problem Set 4 - Due Oct. 27

You should read chapters 8-10 of the book. As usual, *problems* are for theorists and extra credit seekers, although everyone should look at them.

1. **Book Problems** 8.3, 8.5, 9.1, *8.7*, *9.3*,

2. **Integrating Out** Consider a theory with Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\chi^2 - \frac{1}{2}M^2\Phi^2 - \frac{g}{2}\chi^2\Phi \tag{1}$$

   where we are interested in the limit that $M \gg m$ and also $M \gg E_{exp}$, the energies at which we are conducting experiments (for example experiments on χ particle scattering).

   First note the Feynman rules for this theory. Compute the equation of motion for the Φ field and write the solution. Next evaluate the Lagrangian on the solution to the Φ equations of motion, and perform an expansion in $1/M^2$. Given the questions of interest, why is it justified to assume $\Box \ll M^2$? What kinds of interactions between χ particles are generated to first and second order in $1/M^2$? Work out the Feynman rules for this new Lagrangian, to second order in $1/M^2$. Finally, compute the scattering amplitude for 2-to-2 scattering of χ particles to second order in $1/M^2$, both using the original Lagrangian and using the new Lagrangian, and check that the results match.

   This procedure is called 'integrating out' a heavy field, and it is an example of the effective field theory philosophy, where we eliminated short distance physics and simply use an effective lagrangian for the long-distance physics (in this case, χ scattering).

3. **LSZ in Action, or Field Redefinition Invariance** First consider the theory of a free massless complex scalar field

$$\mathcal{L} = \partial^\mu\phi^\dagger\partial_\mu\phi \tag{2}$$

   Quantize this field (note that there are two kinds of creation and annihilation operators, which is easiest to understand if we write ϕ using two real scalar fields and then recombine them to make the complex field).

   Now write

$$\phi(x) = (v + \rho(x))e^{i\frac{\theta(x)}{v}} \tag{3}$$

   where v is a real constant and $\rho(x)$ and $\theta(x)$ are both real scalar fields. Work out the Lagrangian in terms of ρ and θ. You should see that it has non-trivial interactions, so quantize ρ and θ and then work out the Feynman rules for their scattering. Finally, compute the scattering amplitude for 2-to-2 scattering of θ particles using these rules, and show that it's zero.

   The lesson is that we can use whatever variables we want to represent the quantum fields – the S-Matrix will be the same, as long as our fields create one-particle states. The form of the Lagrangian doesn't matter at all; it is not a physical observable.

4. **DIY Spontaneous Symmetry Breaking** Consider a theory with Lagrangian

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi - \mu^2 |\phi|^2 - \lambda |\phi|^4 \tag{4}$$

Solve the classical equations of motion for constant $\phi = v$ when the parameter $\mu^2 < 0$. You should find that the re-phasing symmetry $\phi \to e^{i\alpha}\phi$ is spontaneously broken, since you have to choose a phase for v. Choose v real and again write

$$\phi(x) = (v + \rho(x)) e^{i\frac{\theta(x)}{v}} \tag{5}$$

What is the spectrum (what are the particles and their masses)? Compute the leading 2-to-2 scattering amplitude for θ particles in the limit that they have low energy. You can use the methodology from problem 1 if its easier.